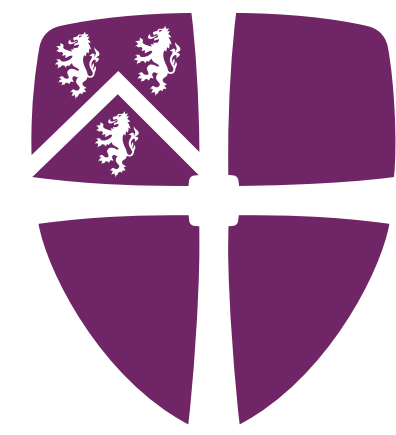


IBM Q

Physics on Quantum Computers

Steve Abel
(*Durham IPPP*)





Durham
University



Related papers ...

S. Abel, N. Chancellor and M. Spannowsky, *Phys.Rev.D* 103 (2021) 1, 016008

S. Abel, M. Spannowsky, *PRX Quantum* 2 (2021) 1, 010349,

S. Abel, A. Blance, M, Spannowsky, *Phys.Rev.A* 106 (2022) 4, 042607

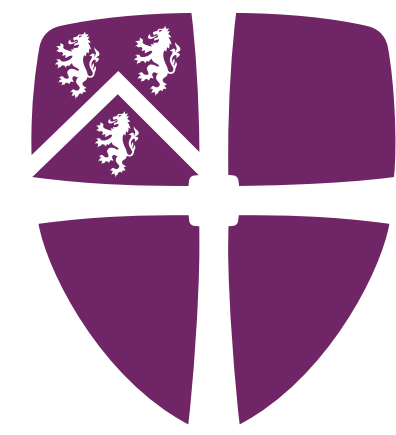
S. Abel, J. Criado, M, Spannowsky, *Phys.Rev.A* 106 (2022) 2, 022601

S. Abel, J. Criado, M, Spannowsky, *Front.Artif.Intell.* 7 (2024) 1368569

S. Abel, M. Spannowsky and S. Williams, Simulating quantum field theories on continuous-variable quantum computers, *Phys.Rev.A* 110 (2024) 1, 1, [arXiv:2403.10619](https://arxiv.org/abs/2403.10619)

S. Abel, M. Spannowsky and S. Williams, Real-Time Scattering Processes with Continuous-Variable Quantum Computers, *Phys.Rev.A* 112 (2025) 1, 012614, [arXiv:2502.01767](https://arxiv.org/abs/2502.01767)

S. Abel, M. Spannowsky and S. Williams, Qumode Tensor Networks for False Vacuum Decay in QFT, [arXiv:2506.17388](https://arxiv.org/abs/2506.17388)



Durham
University



- Motivation: what makes quantum systems so hard?
- Enter Quantum Computing (QC)
- Towards QFT on Continuous Variable QCs
- Qumode lattice on Tensor Networks

1: Motivation

*“Nature is quantum [...] so if
you want to simulate it, you
need a quantum computer”*
- Feynman (1982)



Beginning of a scientific journey that accelerated in recent
years tremendously....

What makes quantum systems so hard?

Let's build a quantum field theory from quantum coins

Classical coins are binary: $T = 1$, $H = 0$



OR



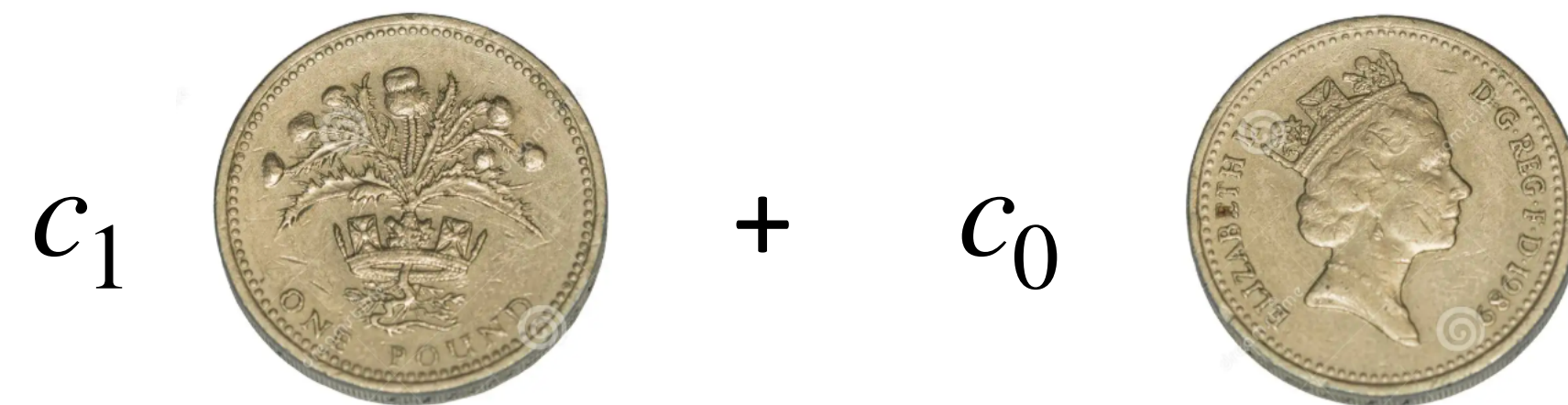
Assumed to be completely independent —

n classical coins — described by n **bits**



n classical coins have 2^n possible states we can enumerate by calling $T = 1$, $H = 0$ so the state is one of the terms in the expansion of $(0 + 1) \times (0 + 1) \times \dots (0 + 1)$

Quantum coins are a linear combination of $T = 1, H = 0$ – “qubit”



The probability of getting H is $|c_0|^2$ and of getting T is $|c_1|^2$
(so e.g. $c_0 = c_1 = 1/\sqrt{2}$ is the most “coin-like”)

Write the state as $|\psi\rangle = c_1|1\rangle + c_0|0\rangle$

Two quantum coins

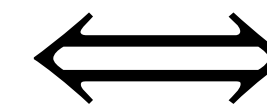
The coins can effect each other. The state has a little bit of T “entangled” with T, H entangled with H, T “entangled” with H, H “entangled” with T.



OR



OR



OR

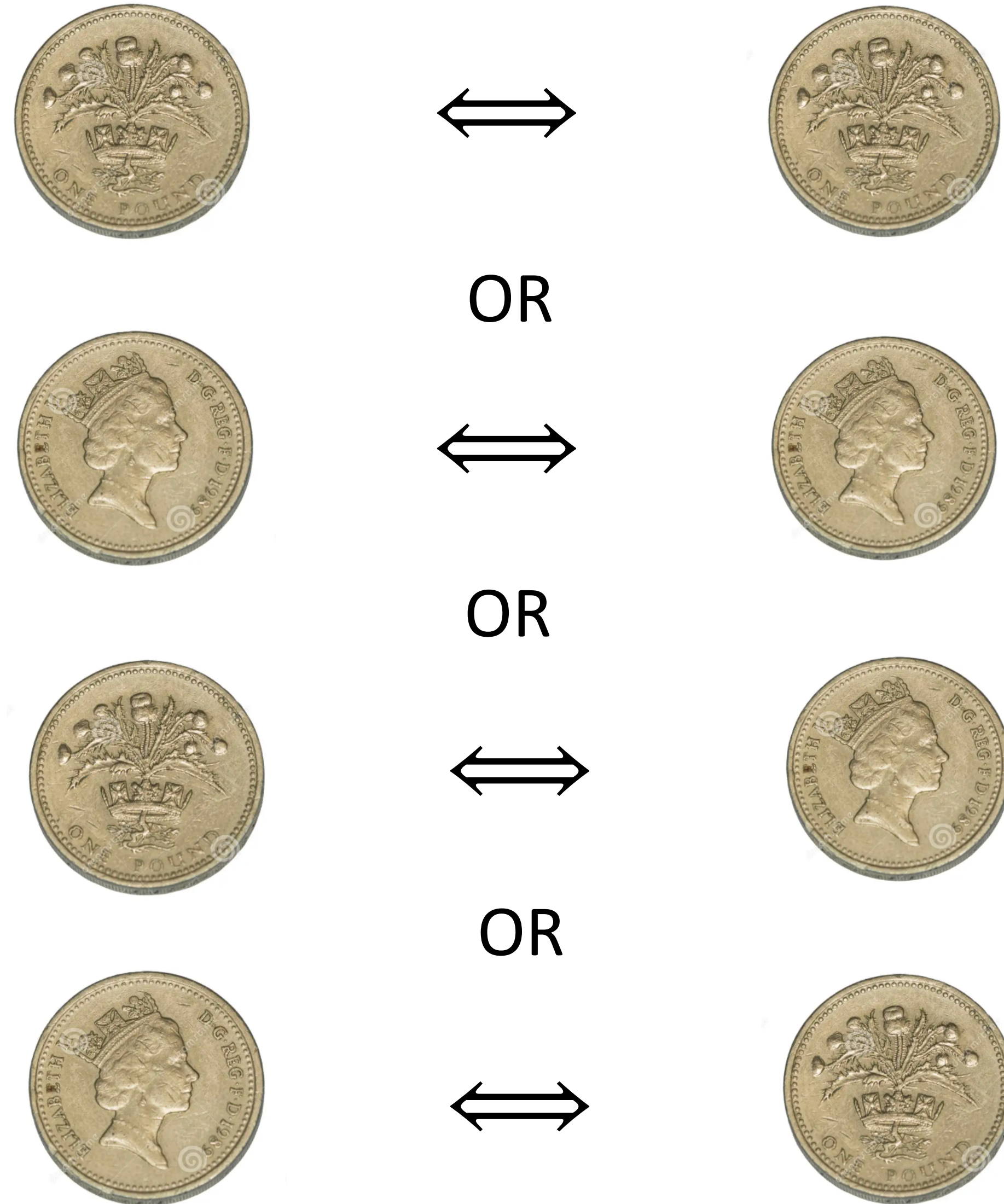


Two quantum coins

The coins can effect each other. The state has a little bit of T “entangled” with T, H entangled with H, T “entangled” with H, H “entangled” with T.

State is described by the amount it contains of the 4 possible outcomes and relations

$$|\psi\rangle = c_{11}|11\rangle + c_{00}|00\rangle + c_{10}|10\rangle + c_{01}|01\rangle$$



Two quantum coins

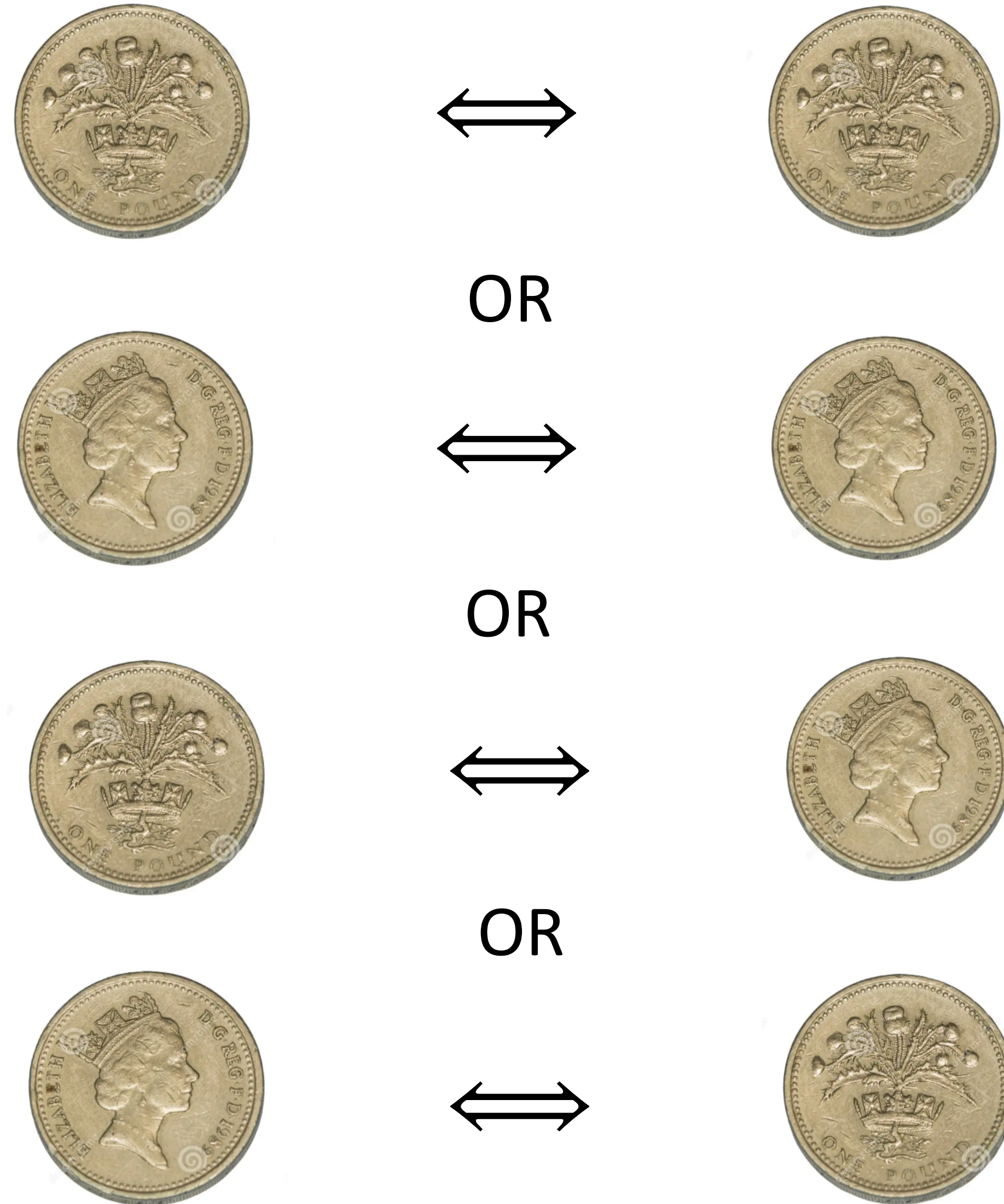
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Coins are **entangled** if we **can't** do this ...

$$|\psi\rangle = (a|0\rangle + b|1\rangle) \times (c|0\rangle + d|1\rangle)$$



Two quantum coins

The coins can effect each other. The state has a little bit of T “entangled” with T, H entangled with H, T “entangled” with H, H “entangled” with T.

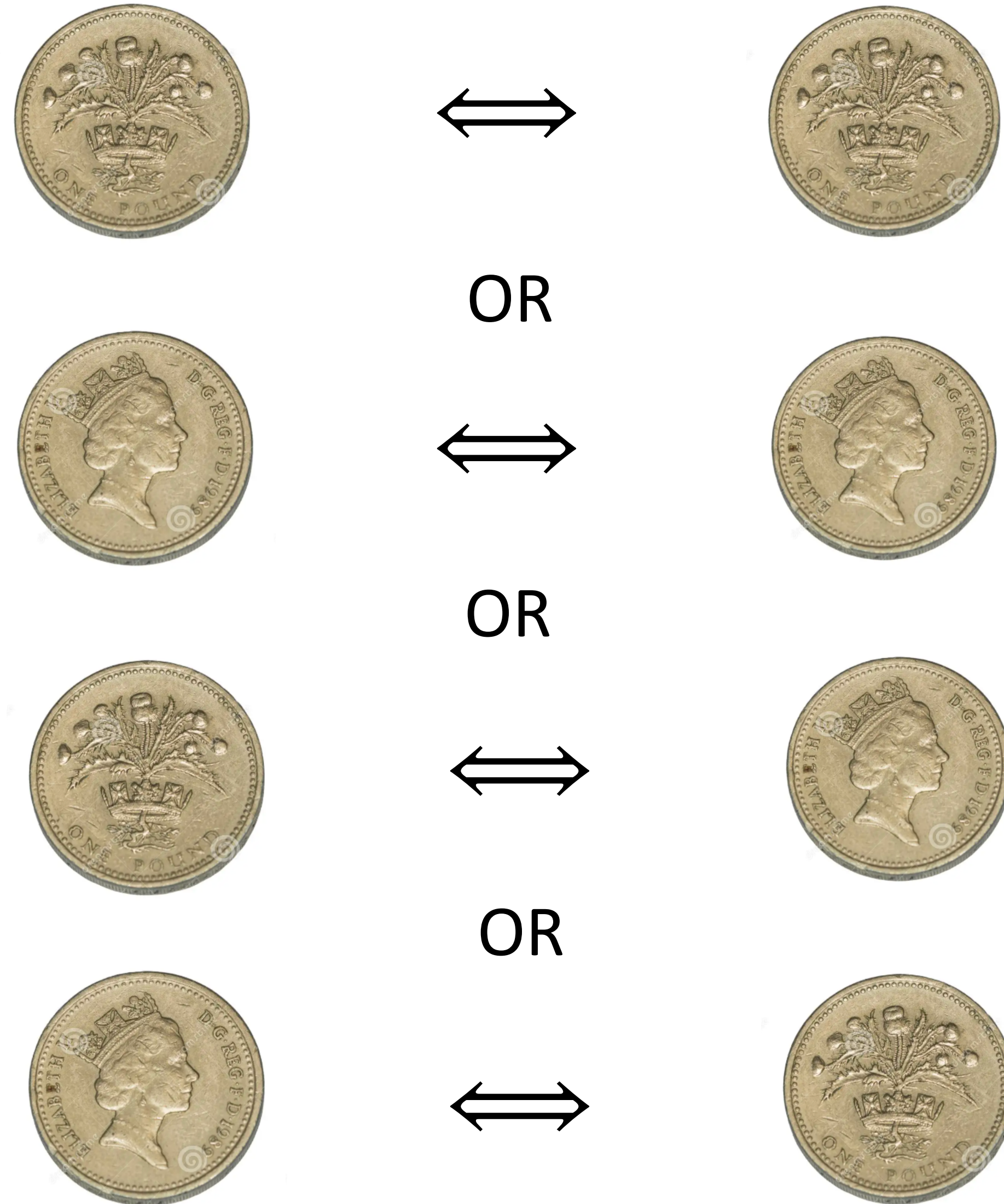
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Coins are **entangled** if we **can't** do this ...

$$|\psi\rangle = (a|0\rangle + b|1\rangle) \times (c|0\rangle + d|1\rangle)$$

For example ... $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



n quantum coins — described by “**qubits**” with a linear combination of 0 (head) or 1 (tail) and entangled many ways



The actual state is now $|\psi\rangle = c_{11\dots 1}|11\dots 1\rangle + c_{01\dots 1}|01\dots 1\rangle + c_{00\dots 1}|00\dots 1\rangle + \dots$

It now has 2^n continuous coefficients

Fundamental physics is built out of **fields**: can think of them as an infinite lattice of similar objects

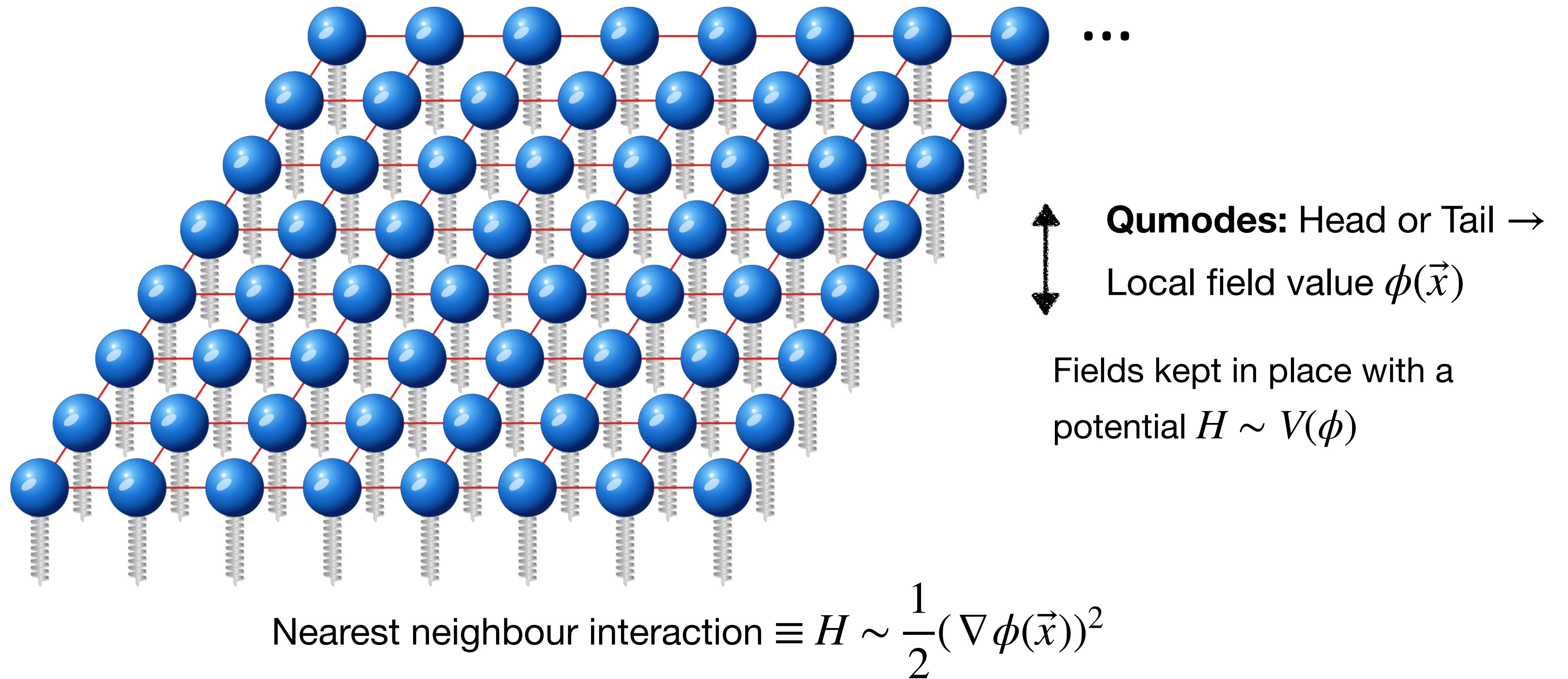


Fundamental physics is built out of **fields**: can think of them as an infinite lattice of similar objects

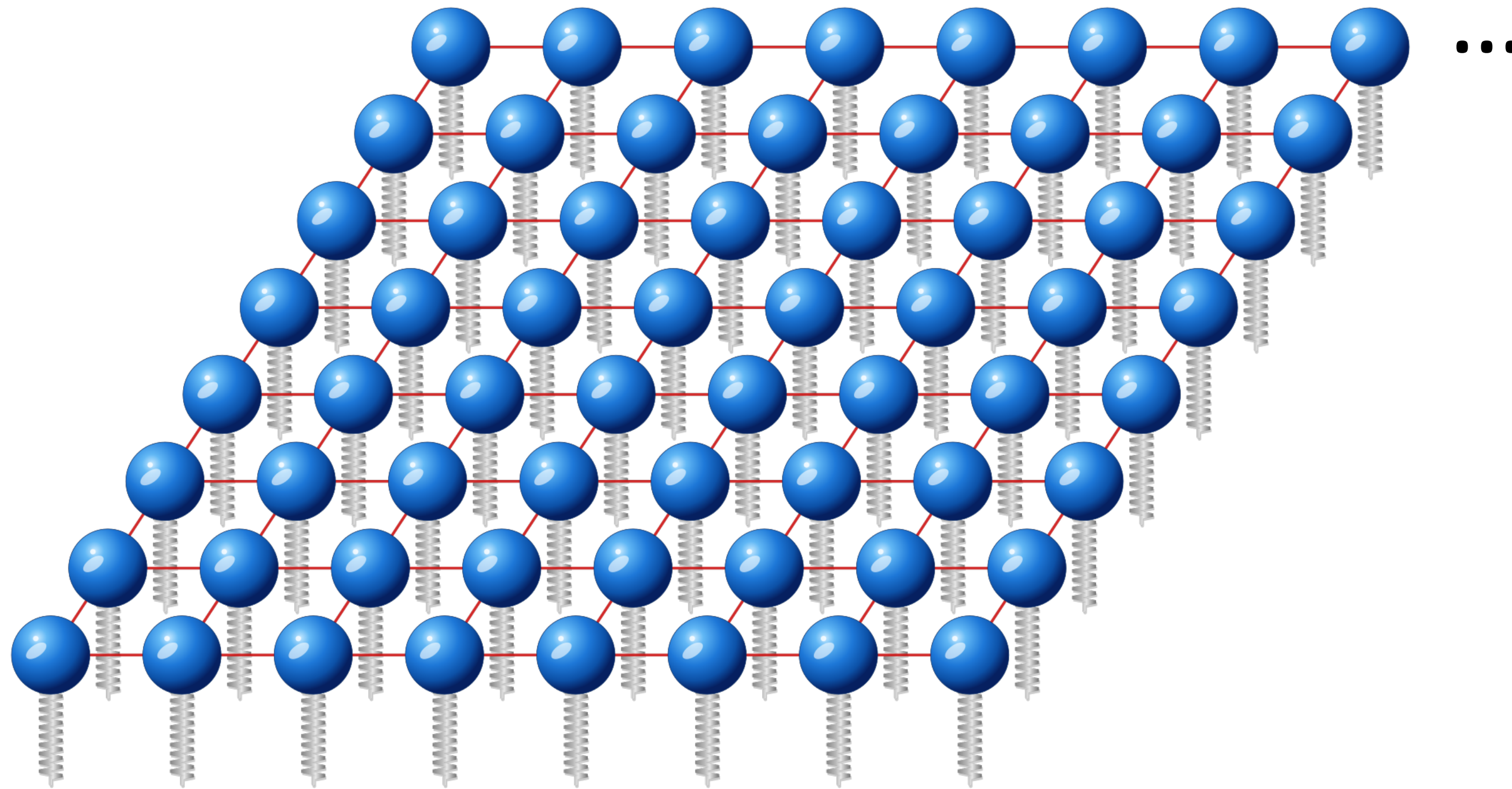


Theory is “local” if “coins” talk to each other only through local (onsite or nearest neighbour) interactions
The field theory built this way out of **qubits** is called the **Ising model**.
Local state is typically not **qubits**, but high dimensional (**qudit**) or continuous (**qumodes**) ...

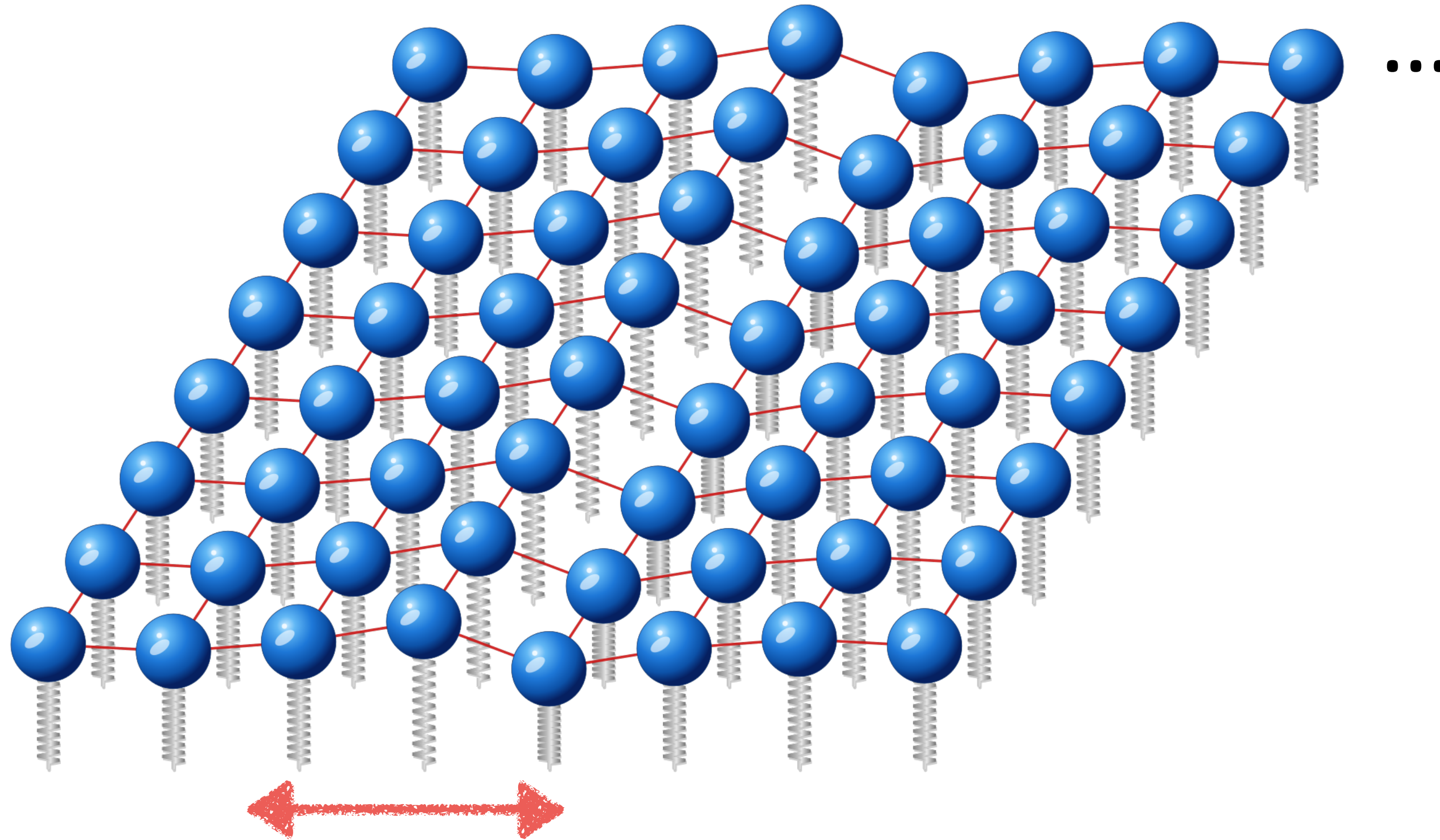
Fundamental physics is built out of **fields**: can think of them as an infinite lattice of similar objects



We can already see which processes are going to be easy and which hard to describe ...

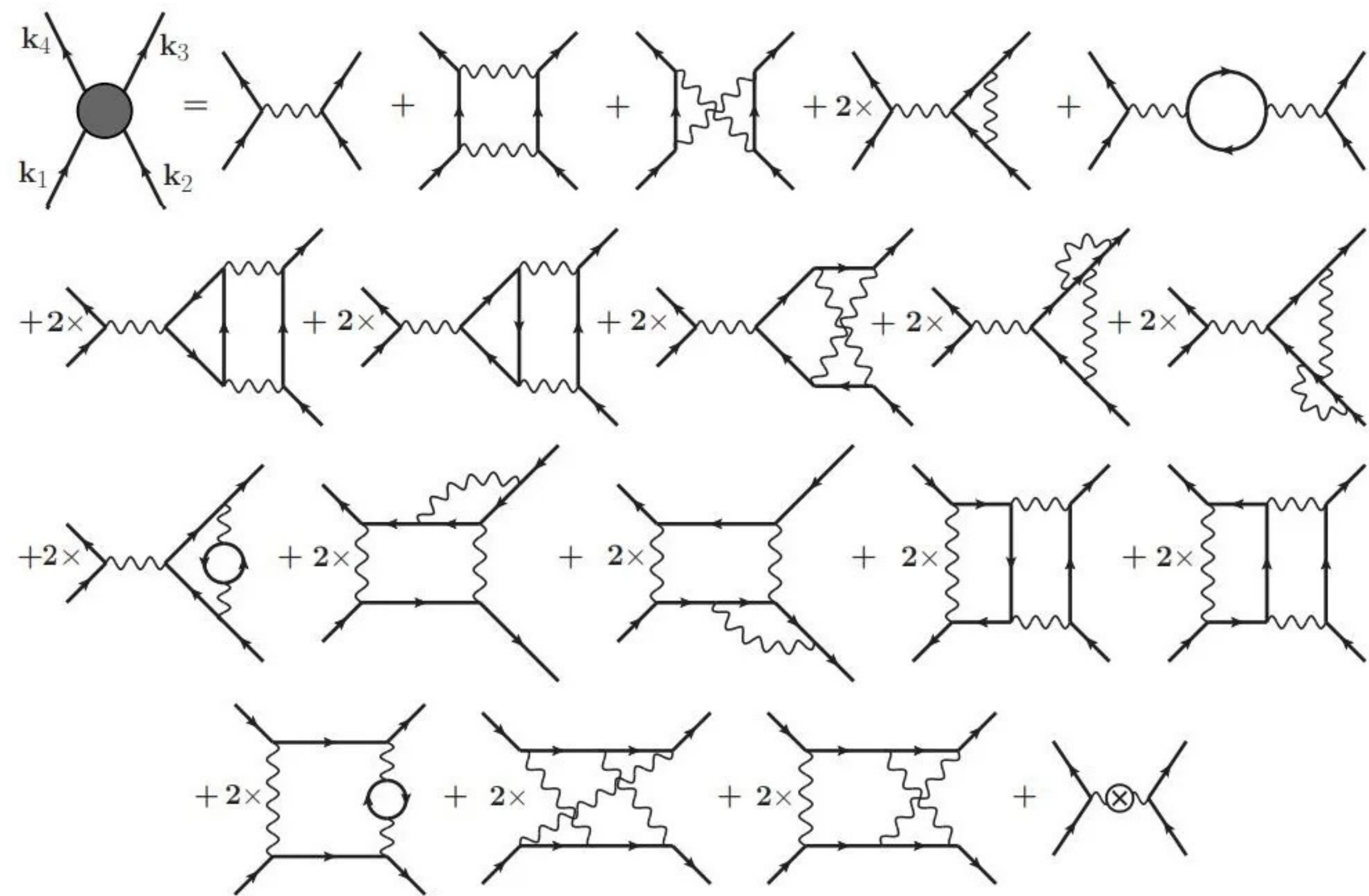


Easy: local oscillations, plane waves etc ... Normal QFT models the *change in entanglement* which is only short range



entanglement short range and important only where wave packets collide

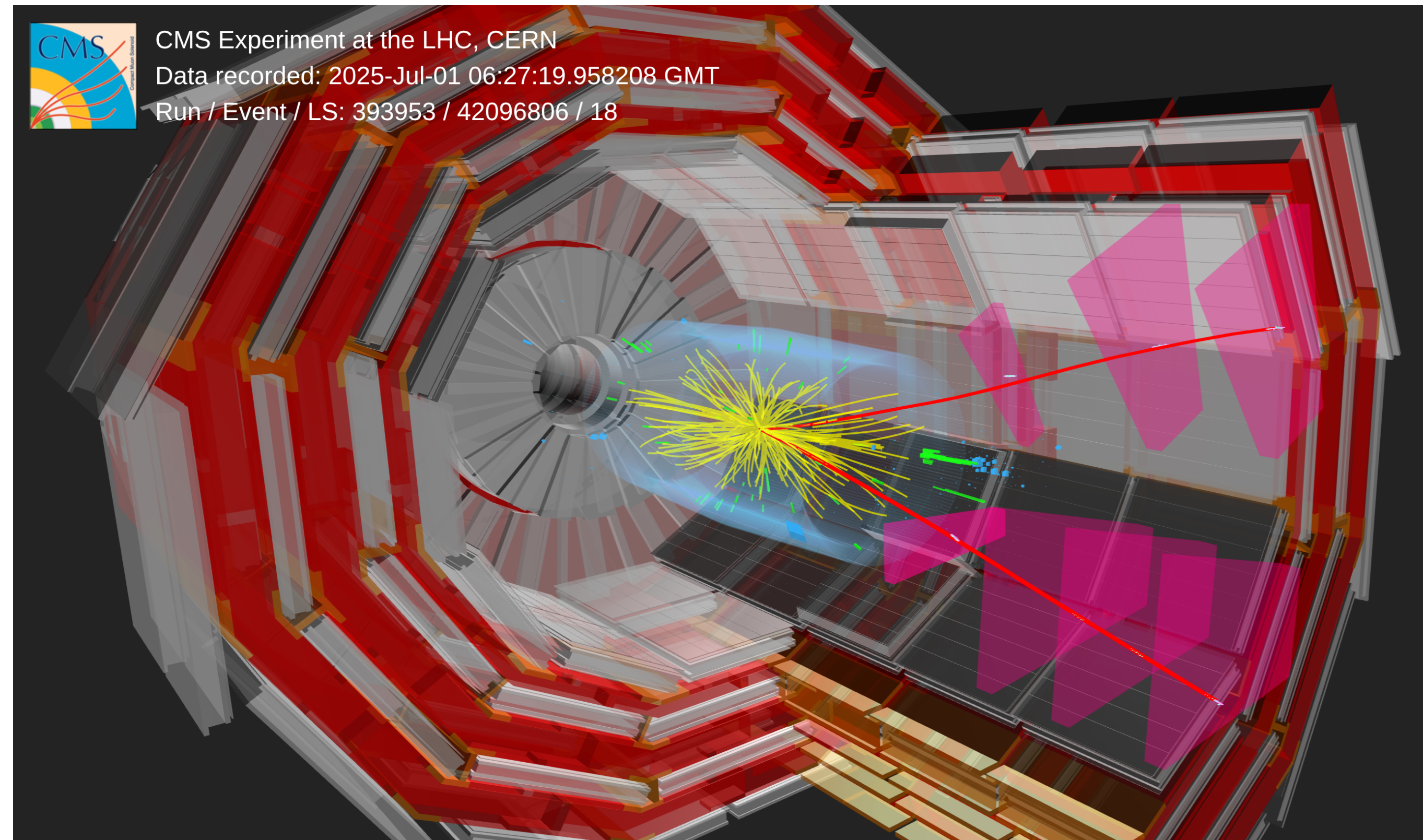
*Morally (i.e. from an entanglement perspective) this incorporates the entire Standard Model and we have useful tools to calculate with — **perturbation theory using path integral methods** treats systems as particles with local interaction vertices*



Feynman diagrams are a mnemonic to enumerate all the mathematical contributions to the entanglement between incoming wave packets which interact in a local interaction region

Away from this region perturbation theory assumes them to be free particles

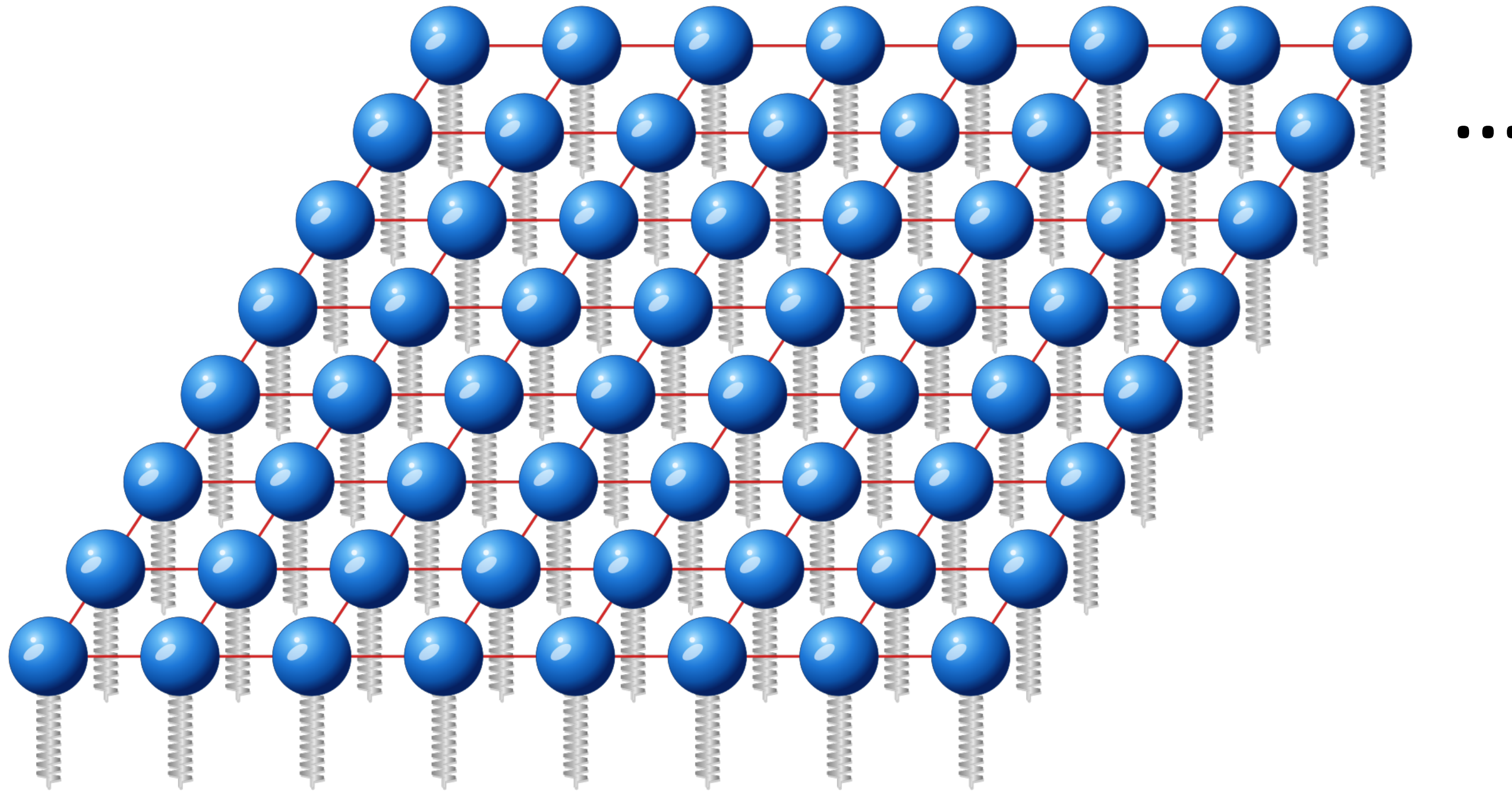
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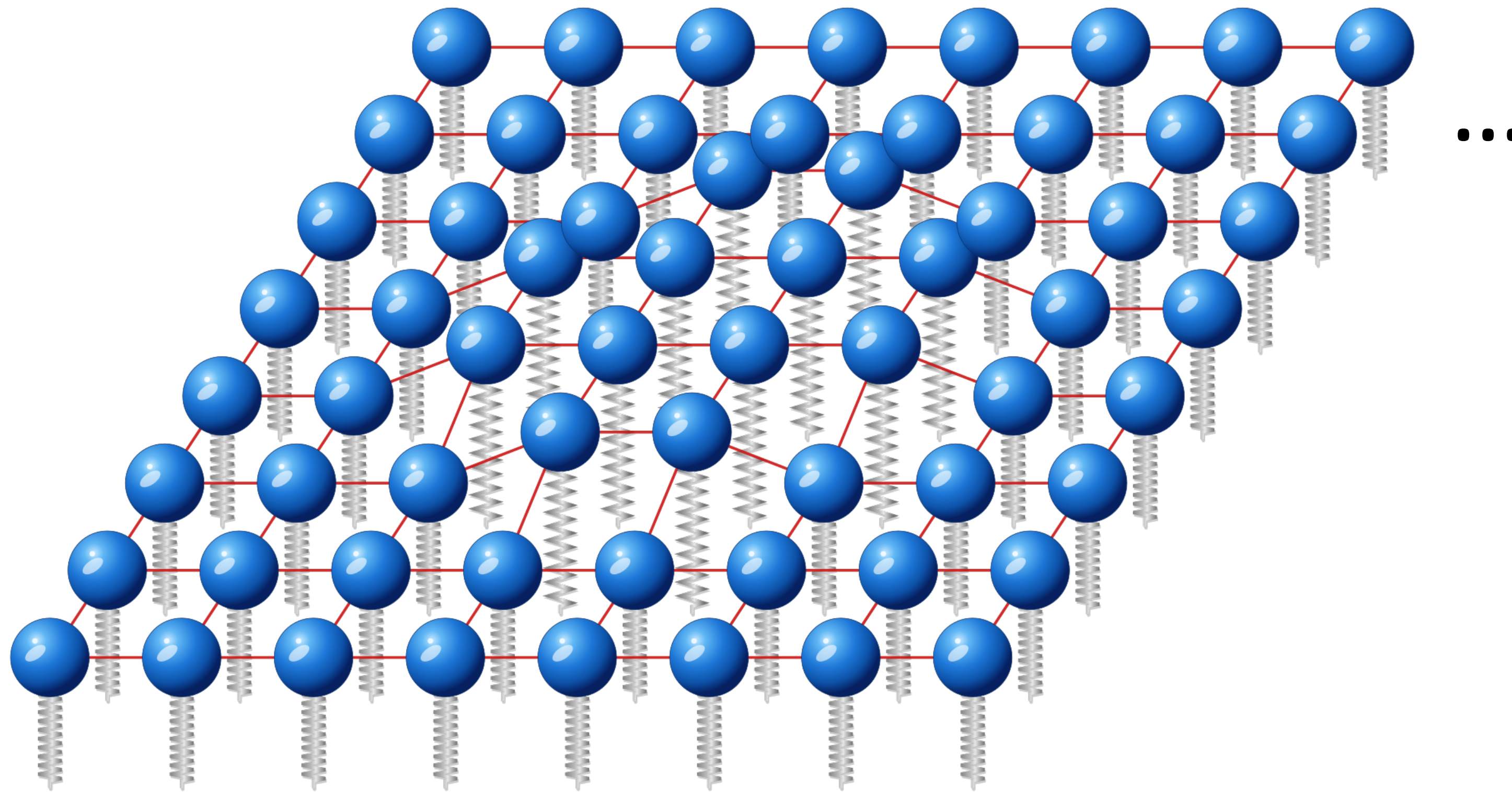
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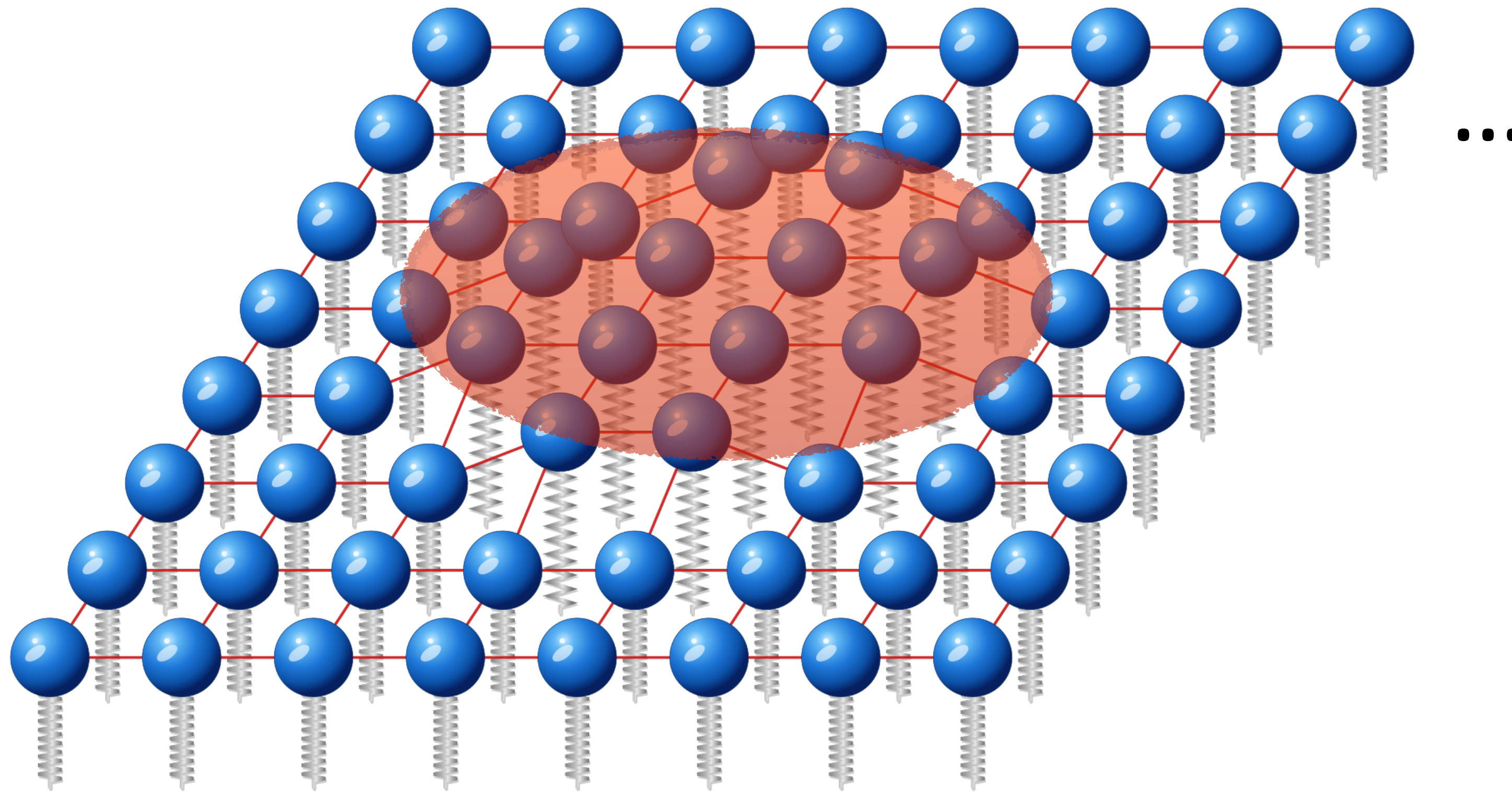
Very difficult: processes that involve coherent field displacements on large scales.



Very difficult: processes that involve coherent field displacements on large scales.

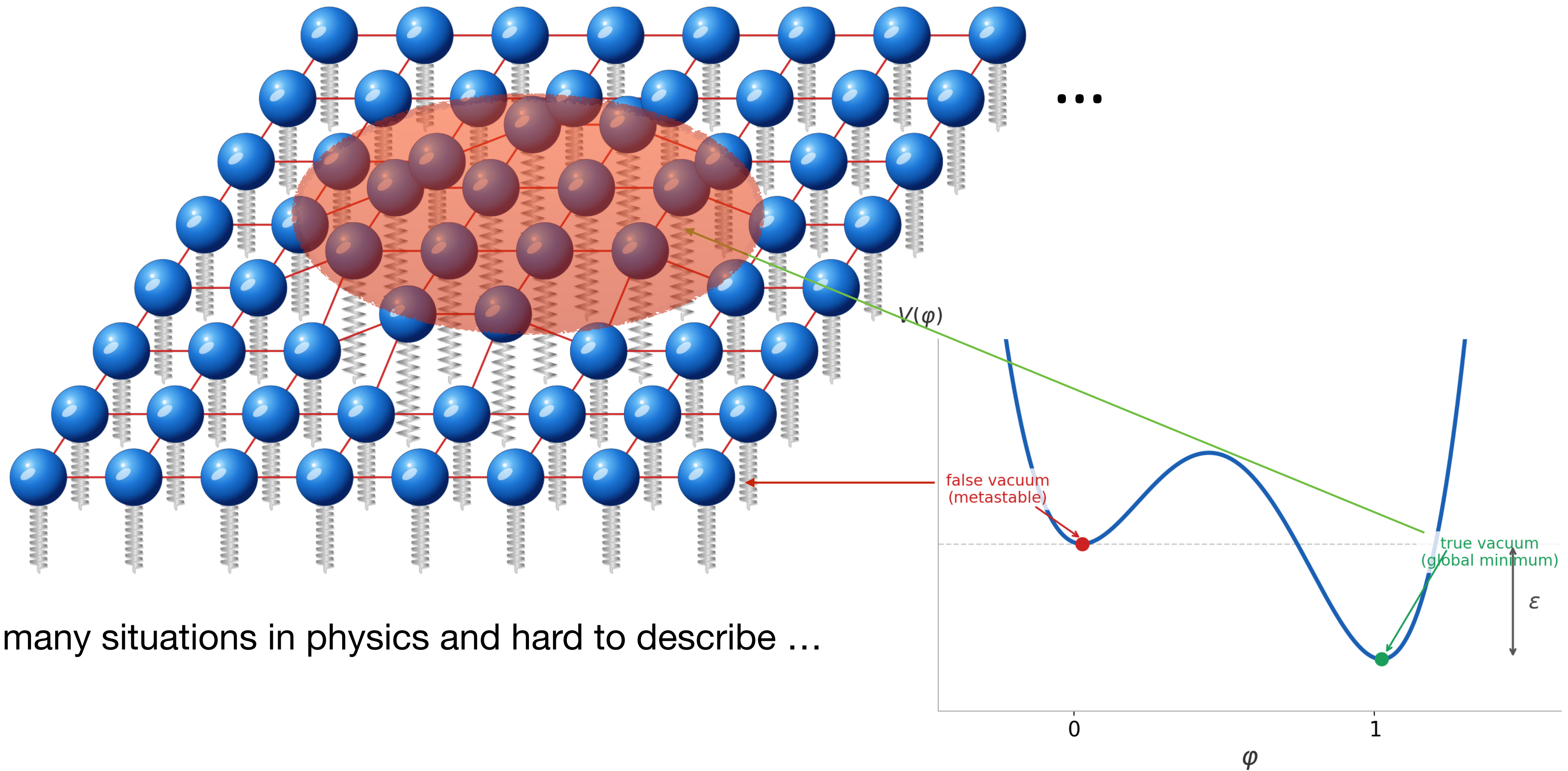


Very difficult: processes that involve coherent field displacements on large scales.



In principle need to describe entanglement across this whole region

Very difficult: processes that involve coherent field displacements on large scales.



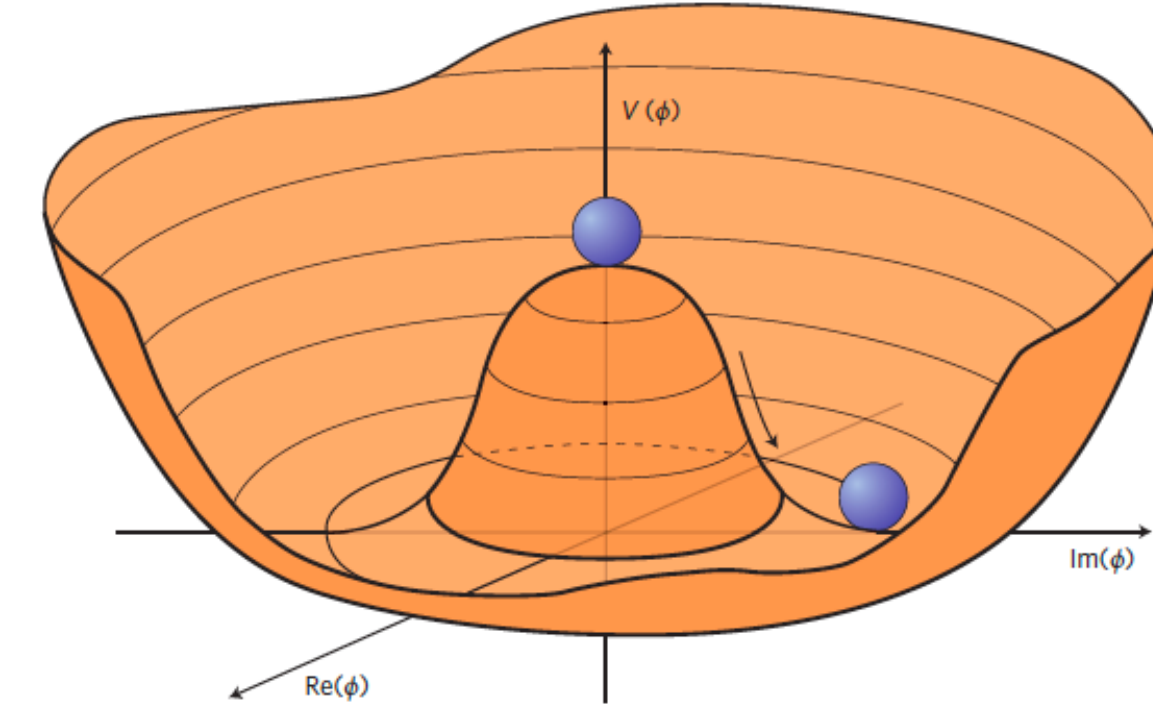
Important for many situations in physics and hard to describe ...

Why classical computation fails — why can't we just do $|\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$ on the whole lattice?

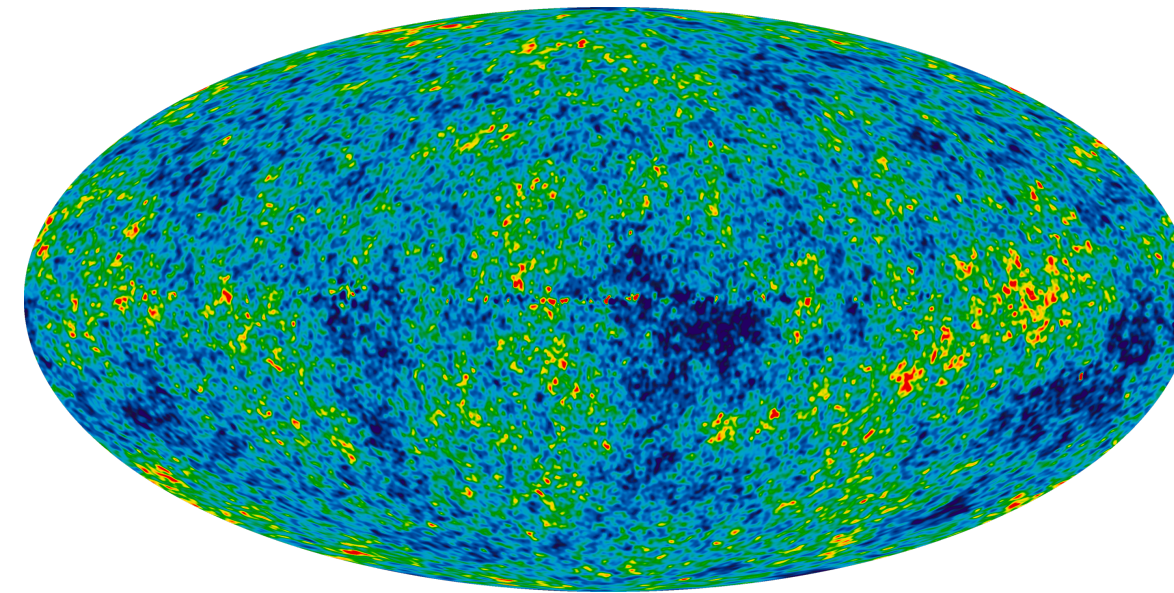
- Suppose our lattice (or our coherent displacement) has N^d sites each with n_ϕ dimensions (qu(d)its).
- Qudits with say $n_\phi = 10$ and a cube $N = 10$ units wide has $n_\phi^{N^d} = 10^{1000}$ coefficients $c_{ijk\dots}$
- This is the true dimensionality of the QFT network if we think of it as a quantum state
- i.e. we need to keep track of 10^{1000} coefficients on a computer
- Fun calculation: computer would turn into a black hole unless its radius is $> 10^{465}m$ (universe is $10^{27}m$)
- Difficulty is because the sites are all coupled/entangled. Otherwise it would be $N^d \times n_\phi$ -dimensional decoupled oscillators
- Well known path integral tricks for tunnelling (e.g. Coleman bounce method) but the “sign problem” is a showstopper for any real dynamics: —

This prevents us fully understanding many important processes ...

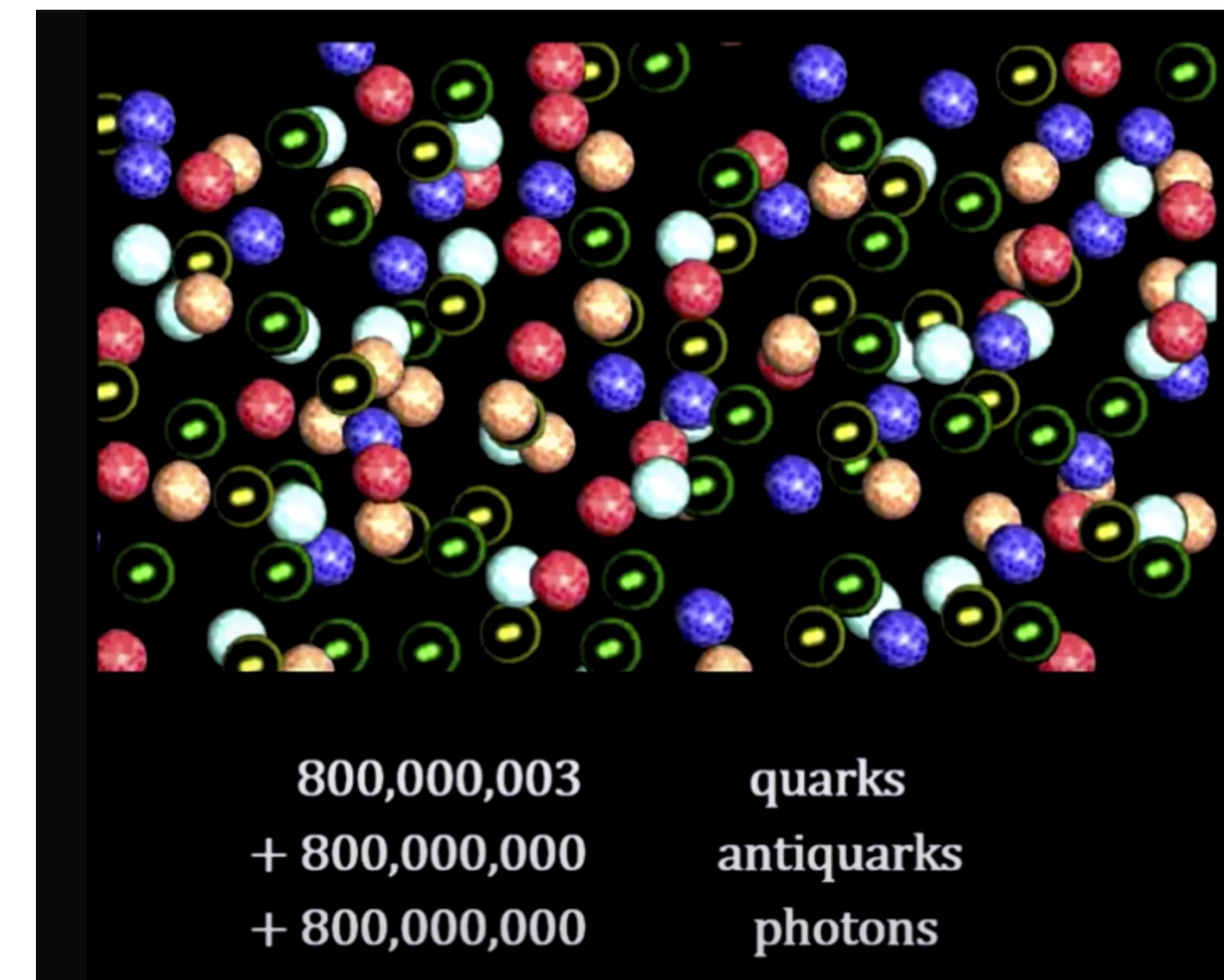
- Electroweak phase transition (Higgs mechanism)



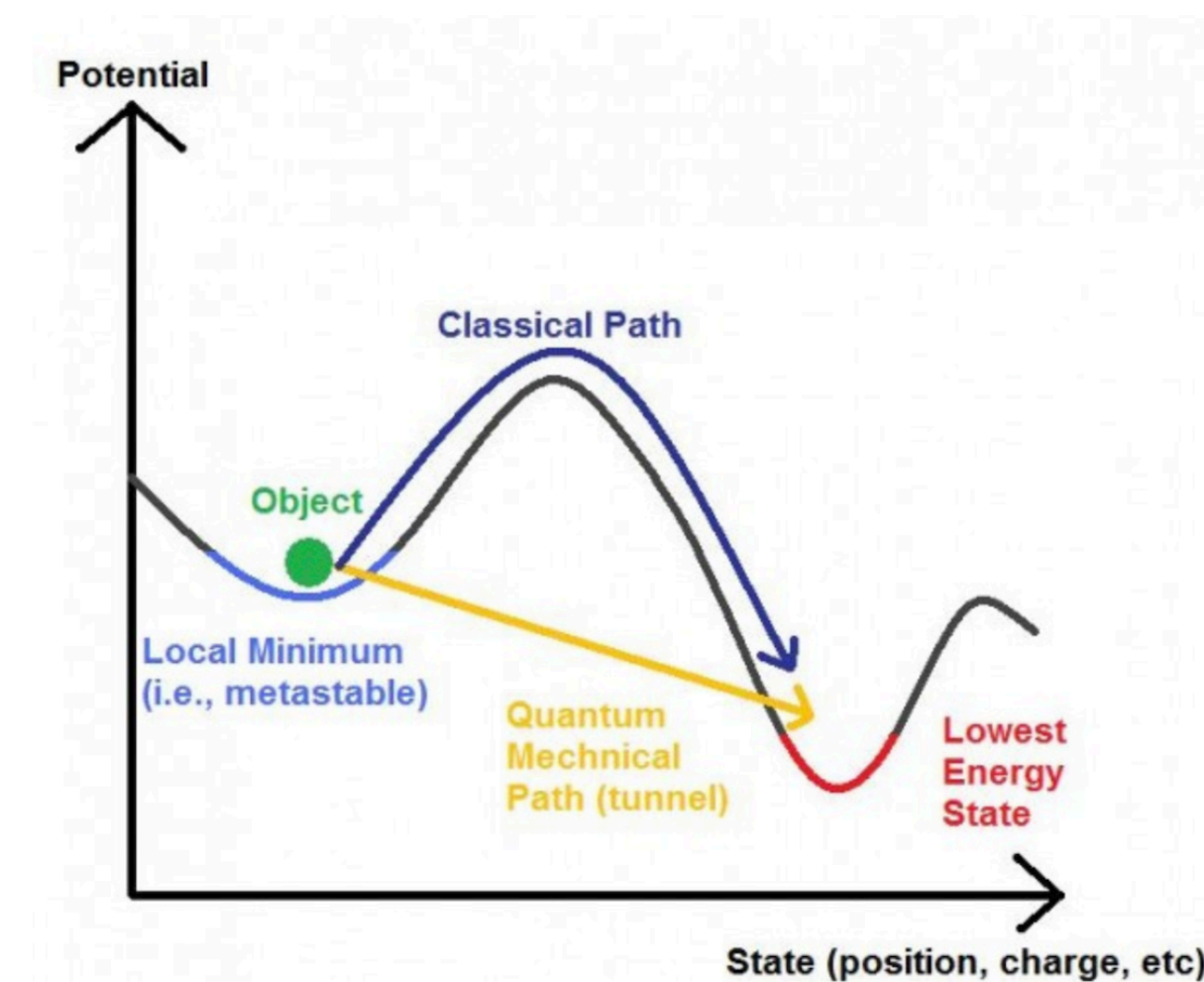
- Inflation



- Baryogenesis (creation of (anti)matter asymmetry)

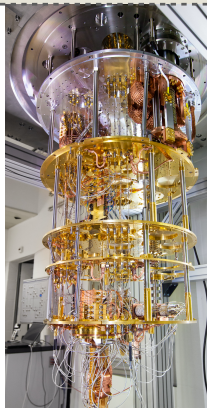
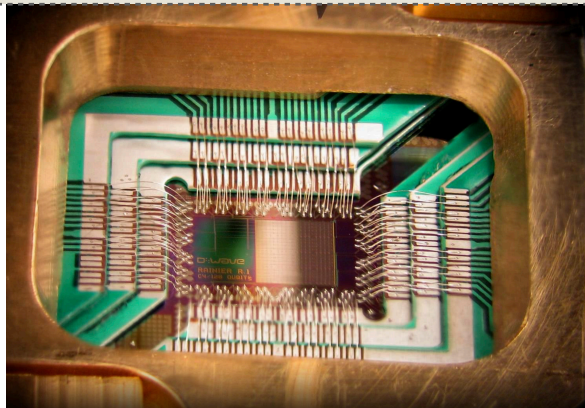
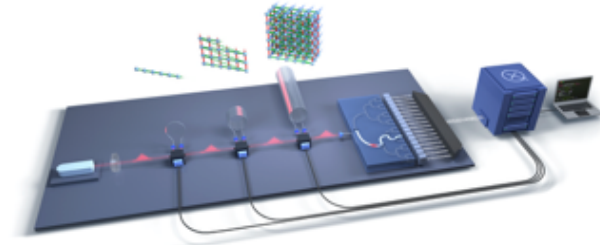
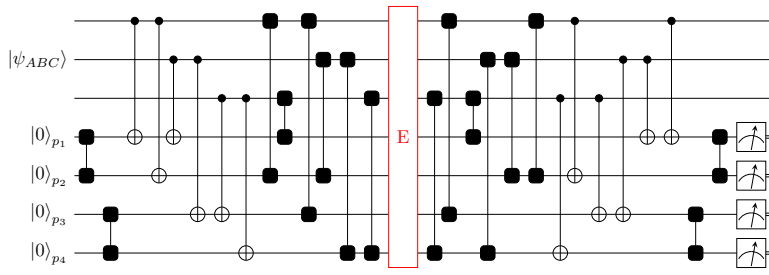
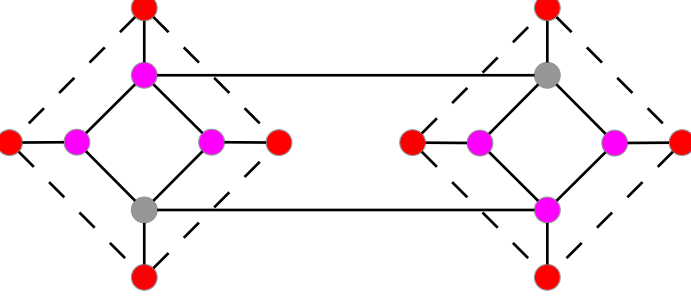
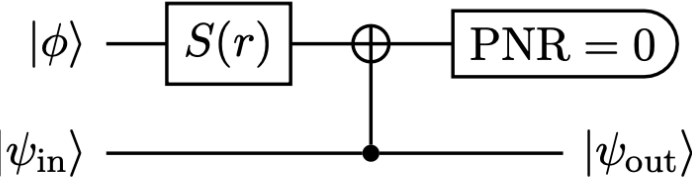


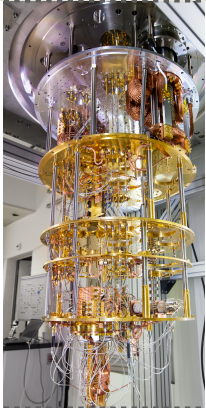
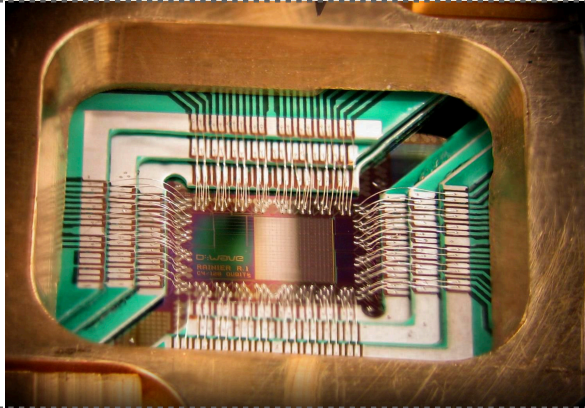
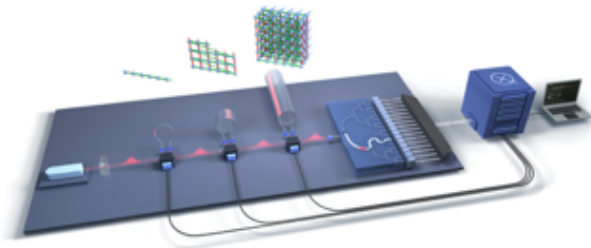
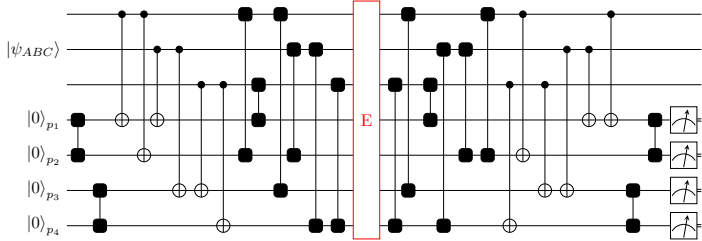
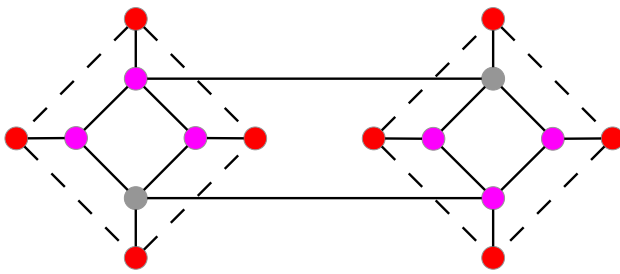
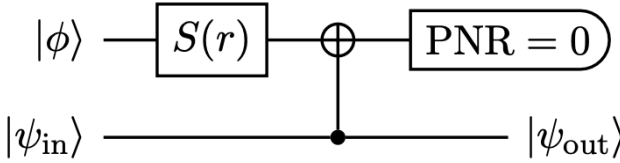
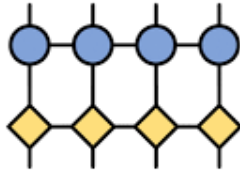
- Instanton processes:

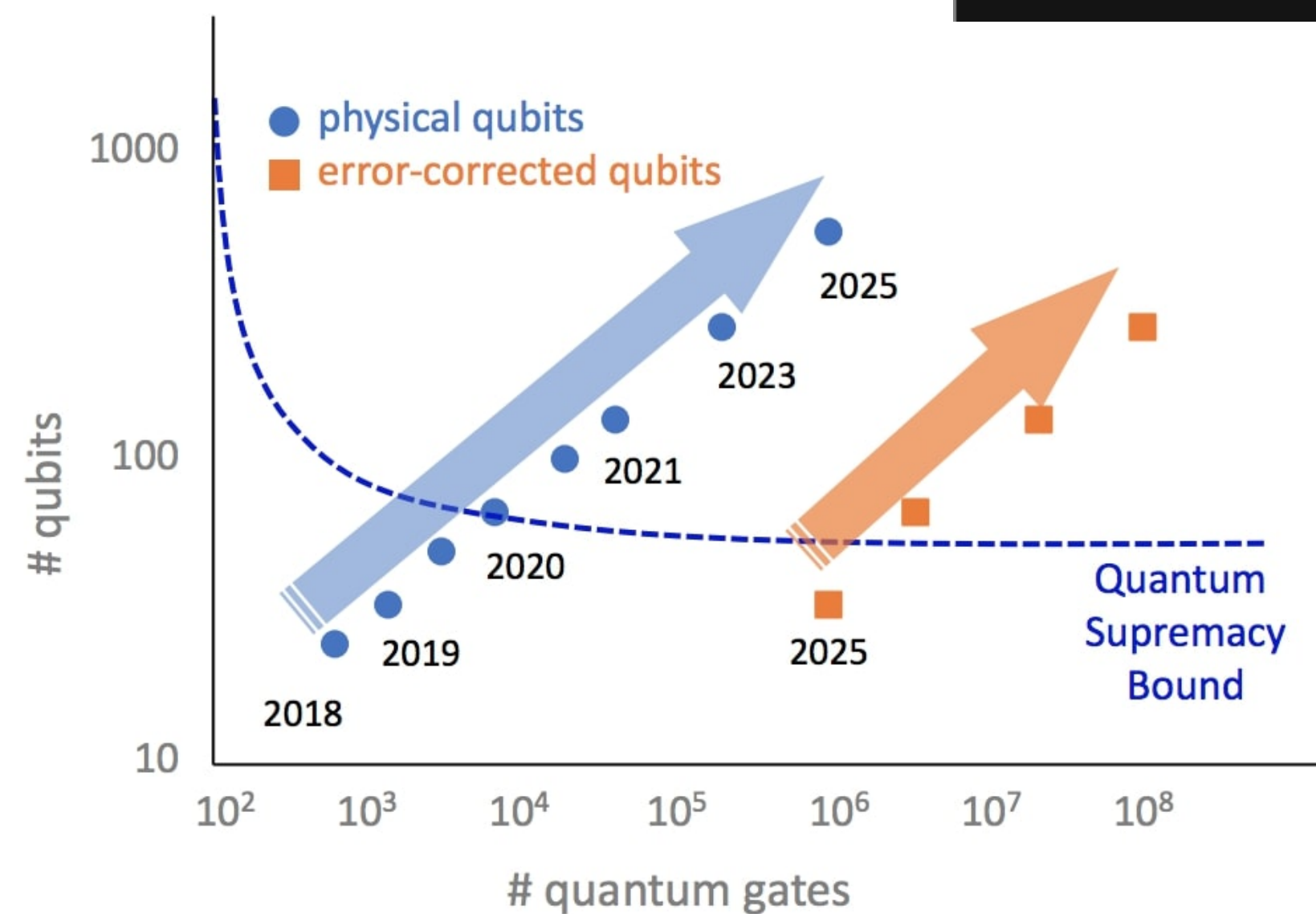
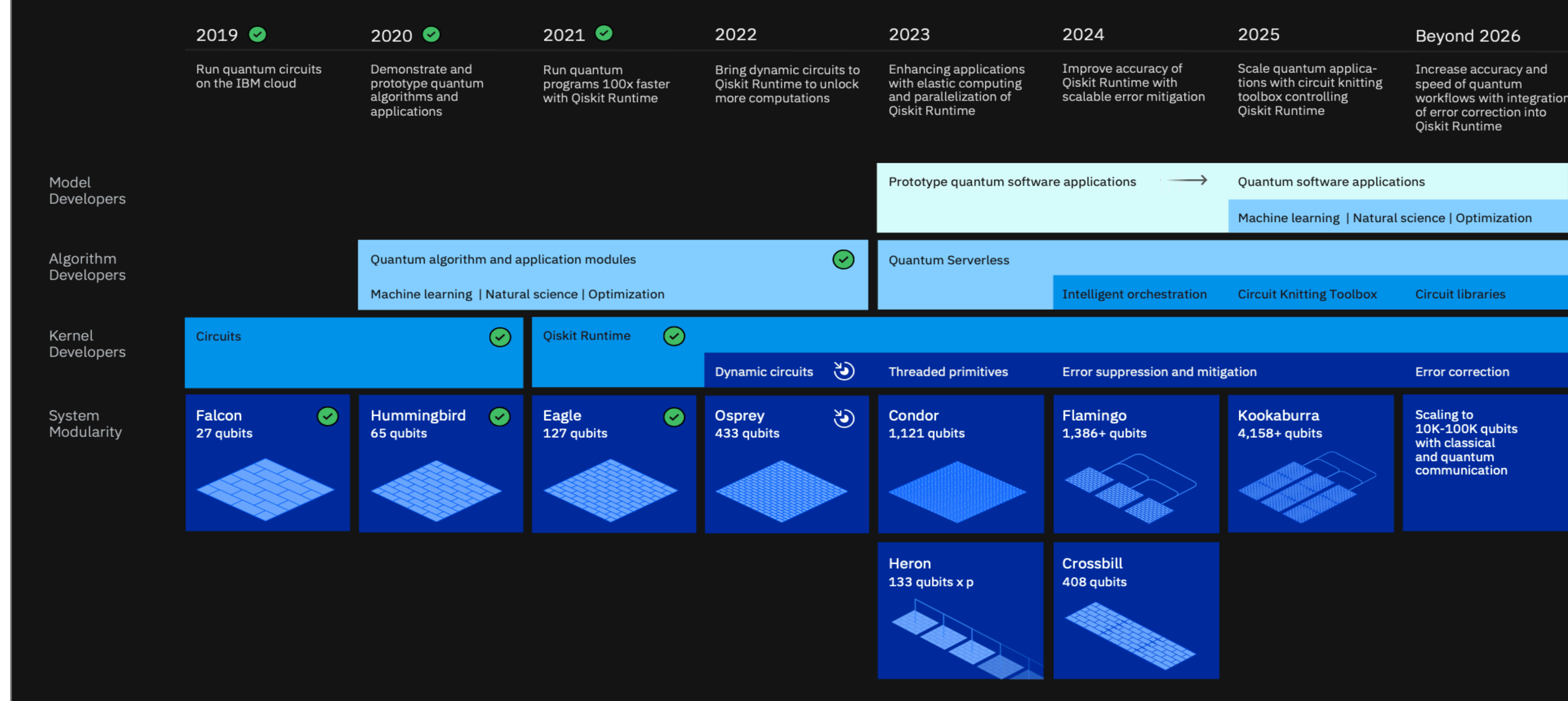


2: Enter Quantum Computing

Feynman '81, Zalka '96,
Jordan, Lee, Preskill ... see
Preskill 1811.10085 for
review.

Type	Discrete Gate	Quantum Annealer	Continuous Variable QC (e.g. photonics)
Property	Universal (any quantum algorithm can be expressed)	Not universal — certain quantum systems	Also universal — G.B.S “quasi-universal”
Where?	IBM - Qiskit ~500 Qubits	DWave - LEAP ~7000 Qubits	Xanadu, ~8 Qumodes - but millions in principle
What?			
Advantage	Most algorithms — lots of support	Continuous time quantum process	Continuous Hilbert spaces (qumodes not qubits)
How?			

Type	Discrete Gate	Quantum Annealer	Continuous variable QC (e.g. photonics)	Tensor-networks
Property	Universal (any quantum algorithm can be expressed)	Not universal — certain quantum systems	Also universal — G.B.S “quasi-universal”	<i>Not quantum:</i> Honourable mention can simulate quantum
Where?	IBM - Qiskit ~500 Qubits	DWave - LEAP ~7000 Qubits	Xanadu, ~8 Qumodes - but millions in principle	ITensor, Tenpy
What?				
Advantage	Most algorithms — lots of support	Continuous time quantum process	Continuous Hilbert spaces (qumodes not qubits)	They exist
How?				



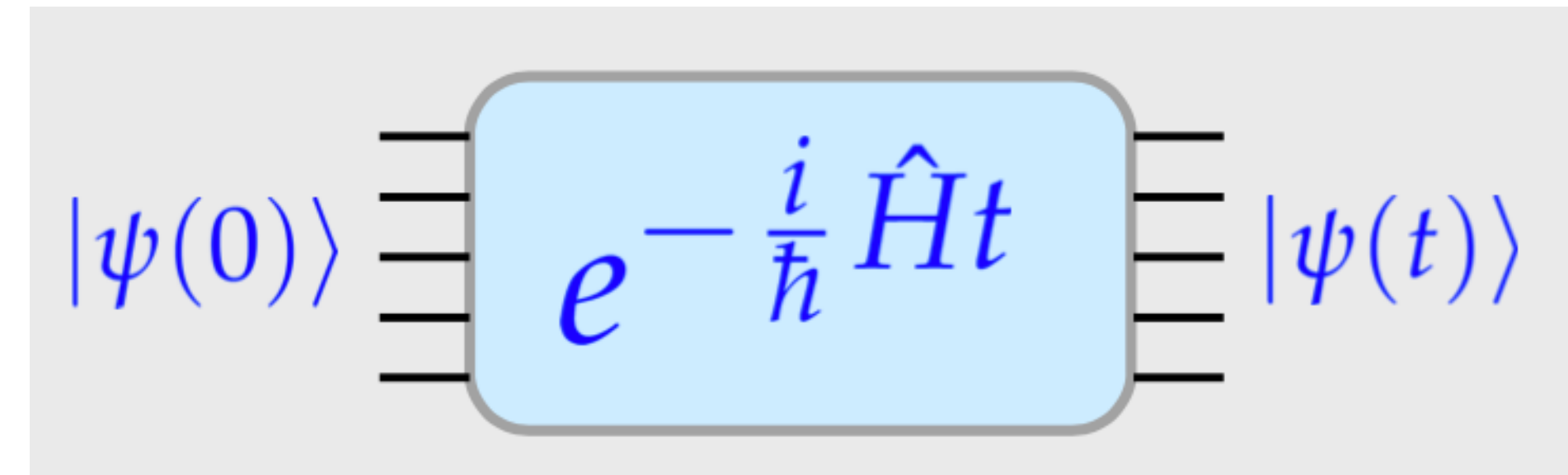
Design Principle

$$Qubits = \sqrt{Gates}$$

10⁶ physical qubits ⇒
1k logical qubits by 2029

For QFT, we would like to be able to do dynamical (Hamiltonian) simulation

- Real-time evolution on quantum computer can avoid sign problem [Kogut, Susskind '74]



- Recall Schrödinger and time evolution equation

$$|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle \quad \rightarrow \quad \frac{\partial \hat{U}(t)}{\partial t} = -iH\hat{U}(t) \rightarrow \hat{U}(t) = e^{-iHt}$$

Trotterization for $H = \sum_k H_k$

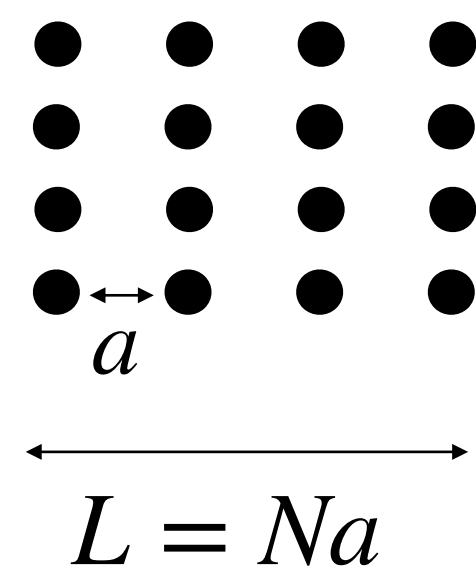
$$U(H, t) = e^{-iHt} \approx \prod_{j=1}^n \prod_k e^{-iH_k \delta t} \quad ; \quad \delta t = t/n$$

Error goes like $\mathcal{O}(\delta t)^2$ for this first order version

Let's compare the required resource with our classical nightmare ...

- Continuous field theories $\phi(x)$ describe particle phenomenology $\longrightarrow$ infinite dimensional matrices

- Discretise $\phi_n(x_k)$



Hilbert space
has dimension

$$\dim H = (n_\phi)^{N^d}$$

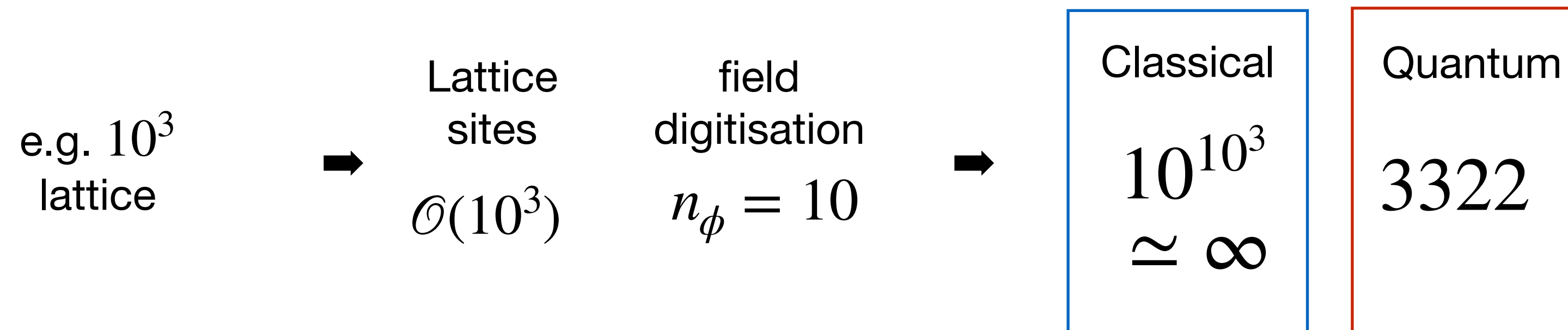
n_ϕ : # of digitised field values — qu(d)its

N : # of lattice points per dim

d : # of dimensions

- On quantum devices algorithms require exp less resource $\ln_2[\dim H]$

[Jordan, Lee, Preskill '12]

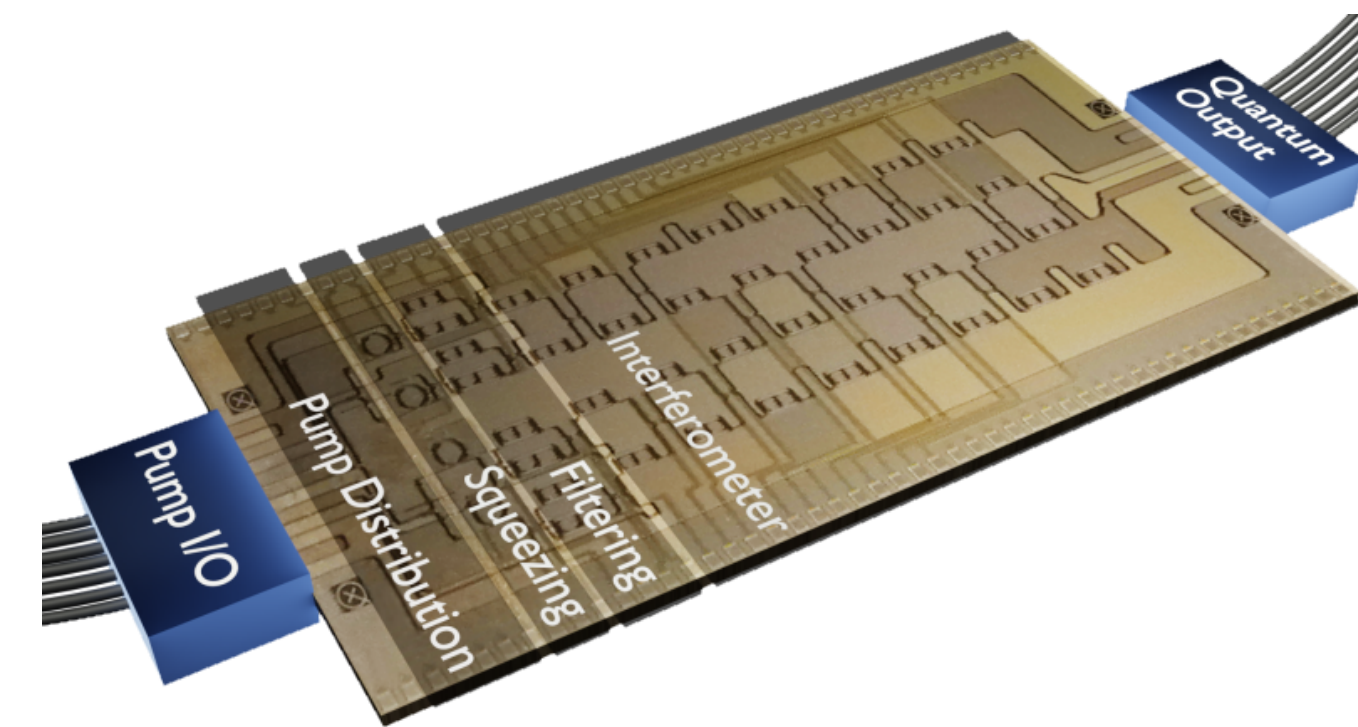


3: Towards QFT on CVQCs

Continuous Variable QC — the most obvious platforms for QFT

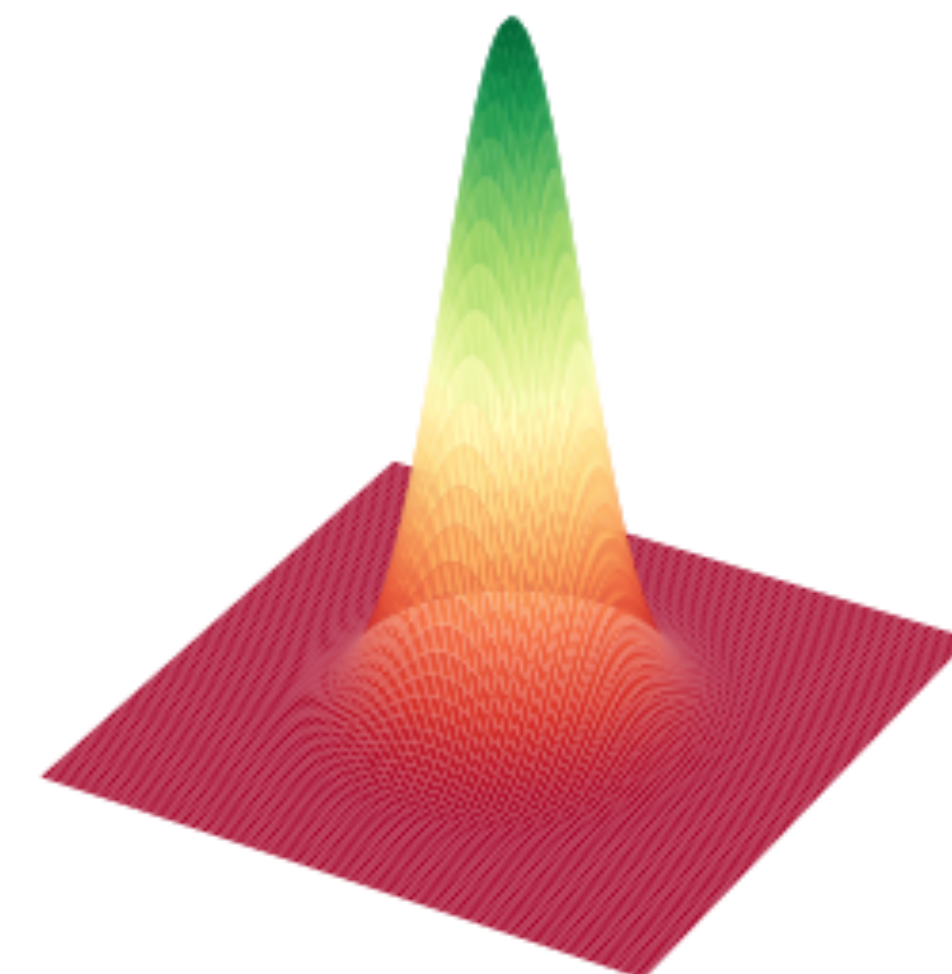
e.g. Photonics. Relies on the manipulation of optical circuits using optical equipment such as interferometers. States can be stored using optical fibres (c.f. RAM) The quantum circuits are defined by the continuous *quadrature variables* (that is the x, p) of quantum harmonic oscillators.

e.g. Borealis and X8 chips of Xanadu,
PsiQuantum, QuiX Quantum, Quandela, Orca, Nu Quantum...



qubit: $|\phi\rangle = \phi_0|0\rangle + \phi_1|1\rangle$

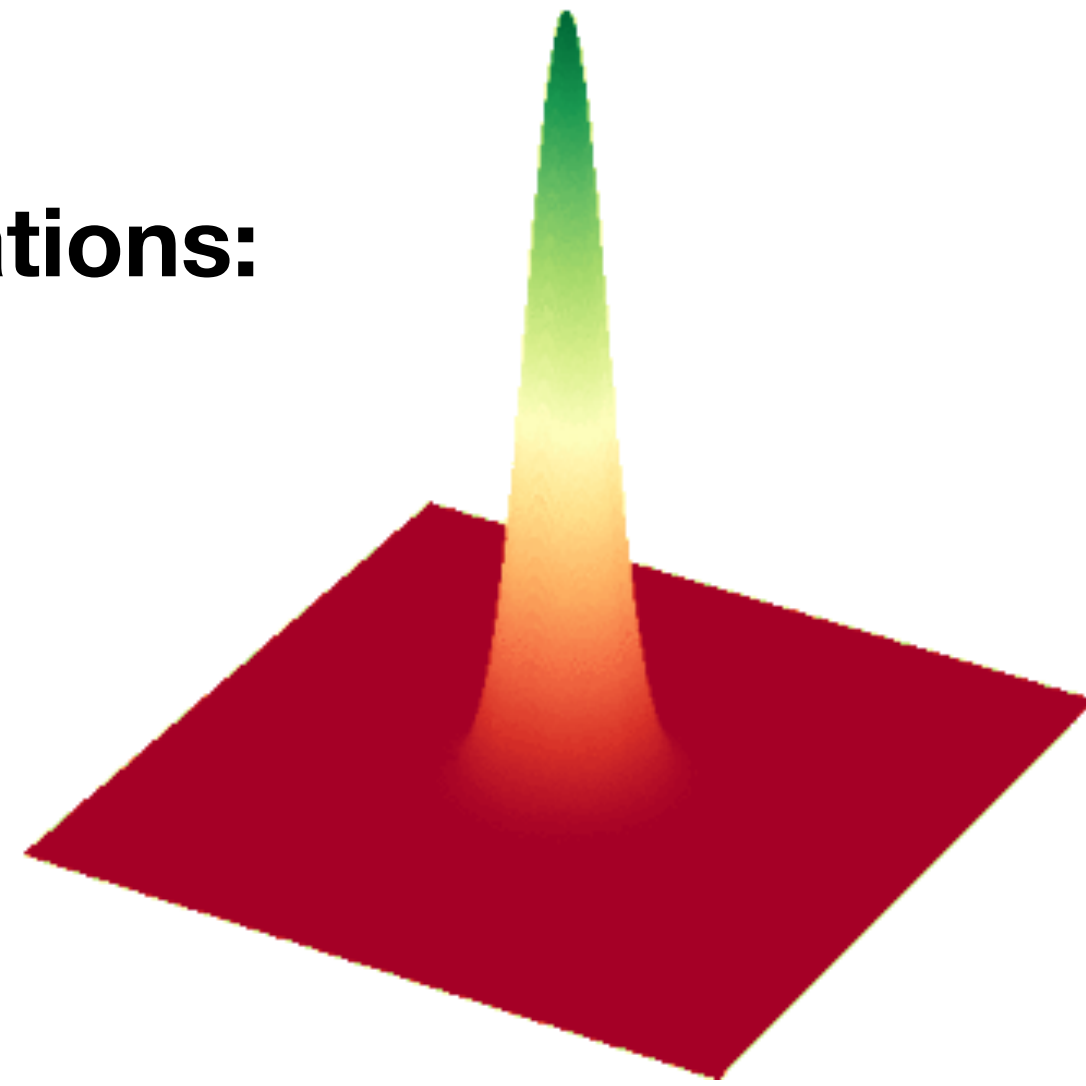
qumode: $|\psi\rangle = \int dx \psi(x) |x\rangle$ (SHO vacuum and excitations)



Basic building block of field theory!! ... Wigner function looks like:

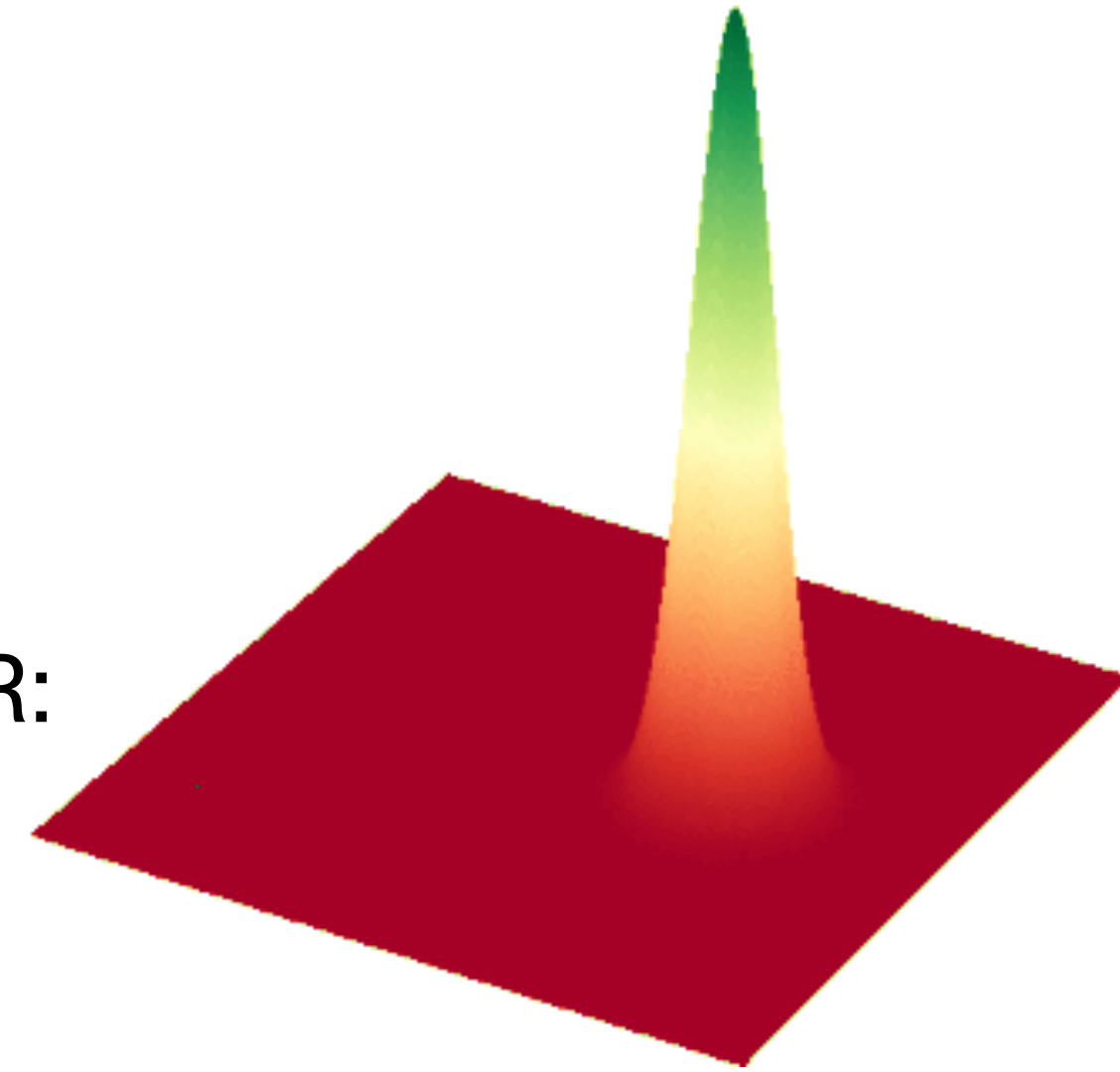
Some simple operations:

Squeezing Gate S:



$$S^\dagger(z)\hat{x}S(z) = e^{-r}\hat{x} ; \quad S^\dagger(z)\hat{p}S(z) = e^r\hat{p} .$$

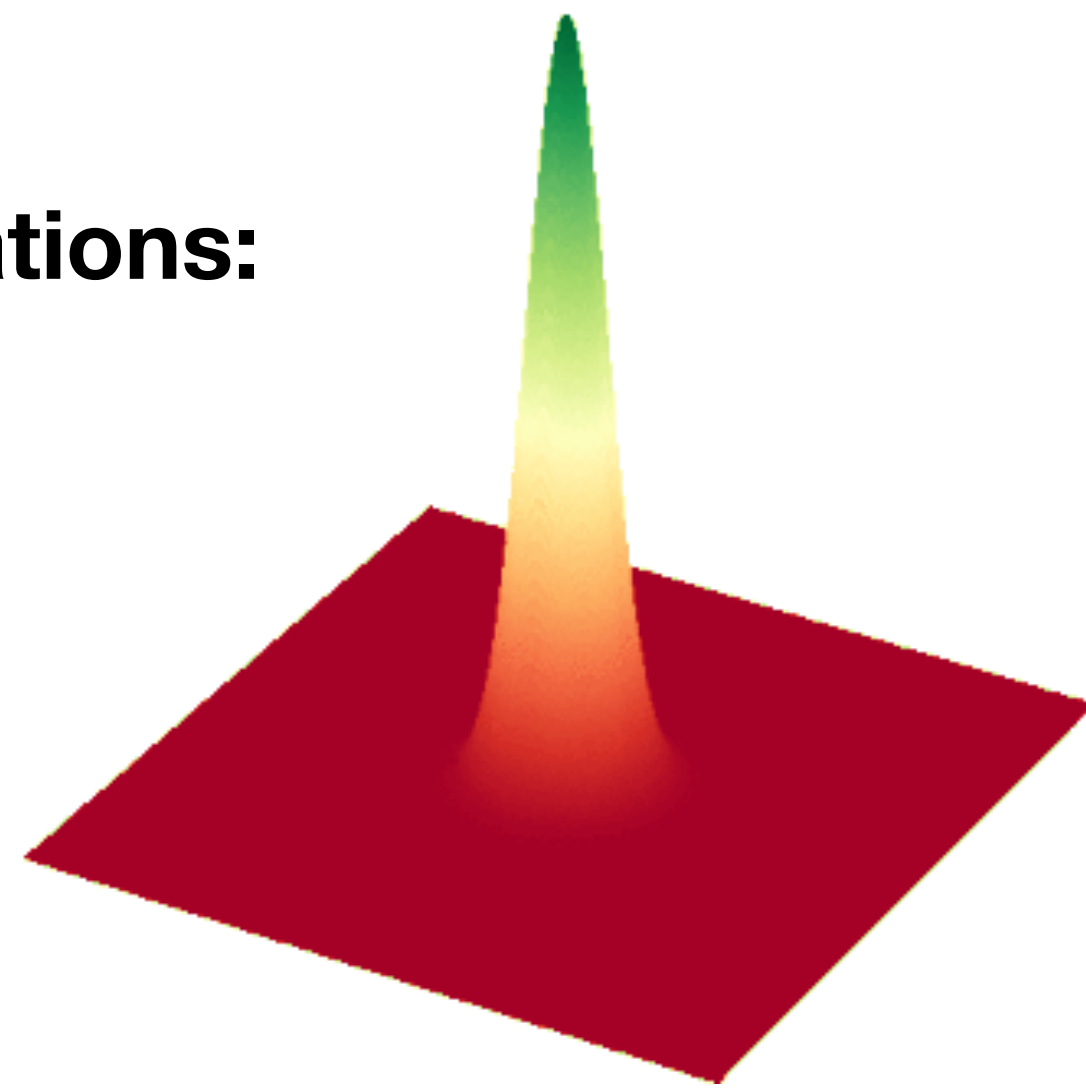
Rotation Gate R:



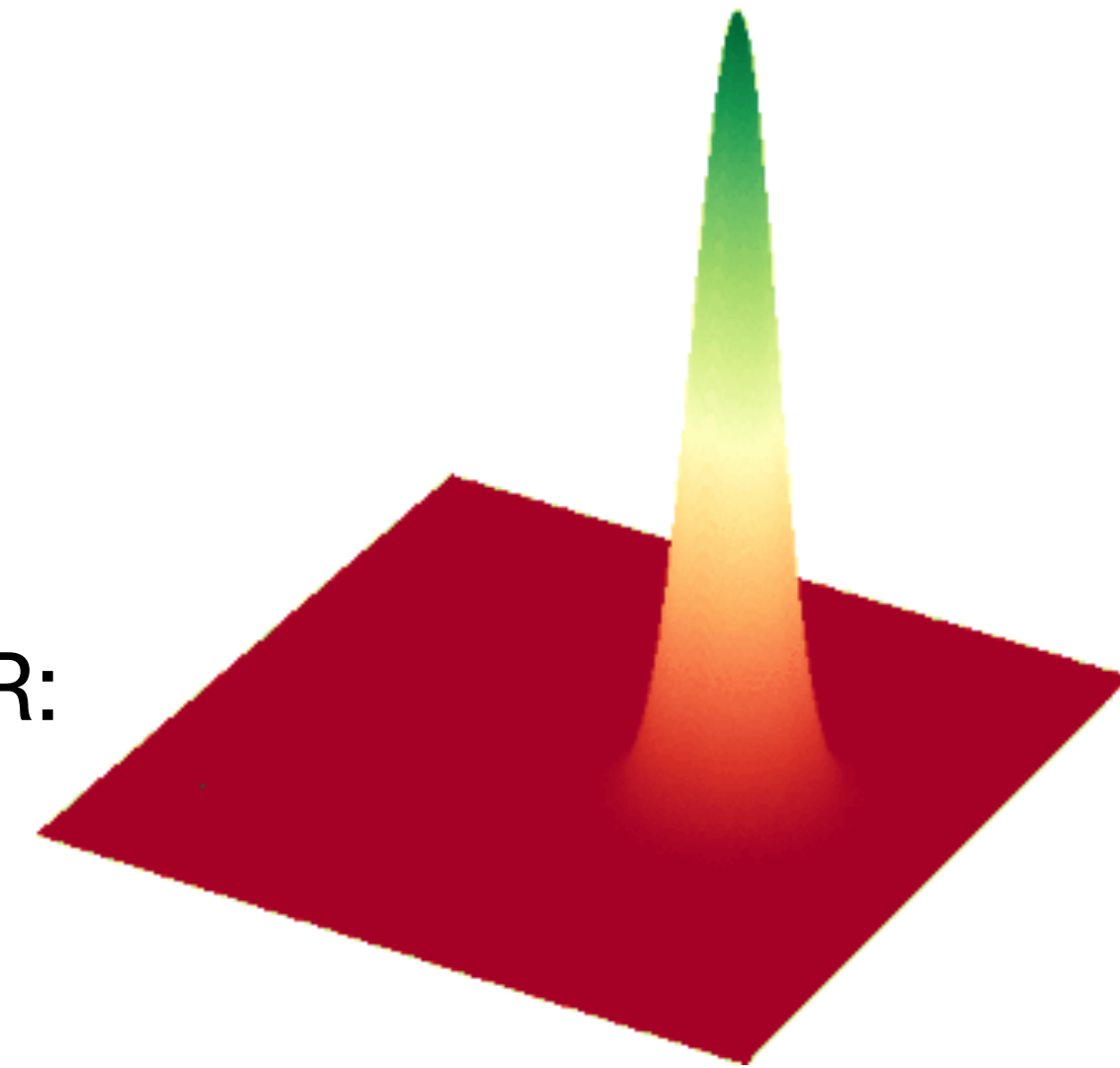
$$R(\theta) = e^{i\theta\hat{a}^\dagger\hat{a}} = e^{i\theta\left(\frac{1}{2}\hat{p}^2 + \frac{1}{2}\hat{x}^2 - \frac{1}{2}\right)}$$

Some simple operations:

Squeezing Gate S:



Rotation Gate R:



$$S^\dagger(z)\hat{x}S(z) = e^{-r}\hat{x} ; \quad S^\dagger(z)\hat{p}S(z) = e^r\hat{p} .$$

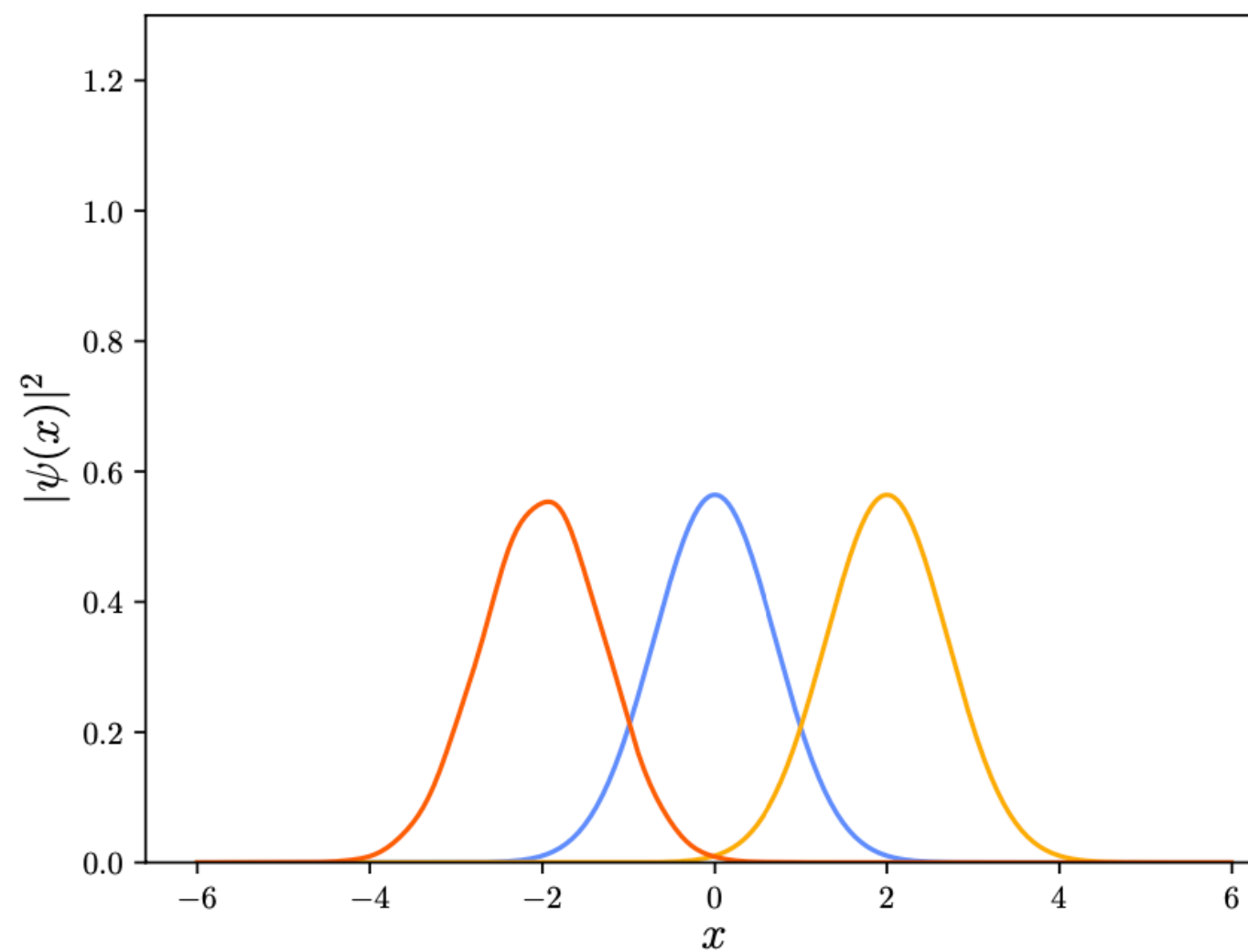
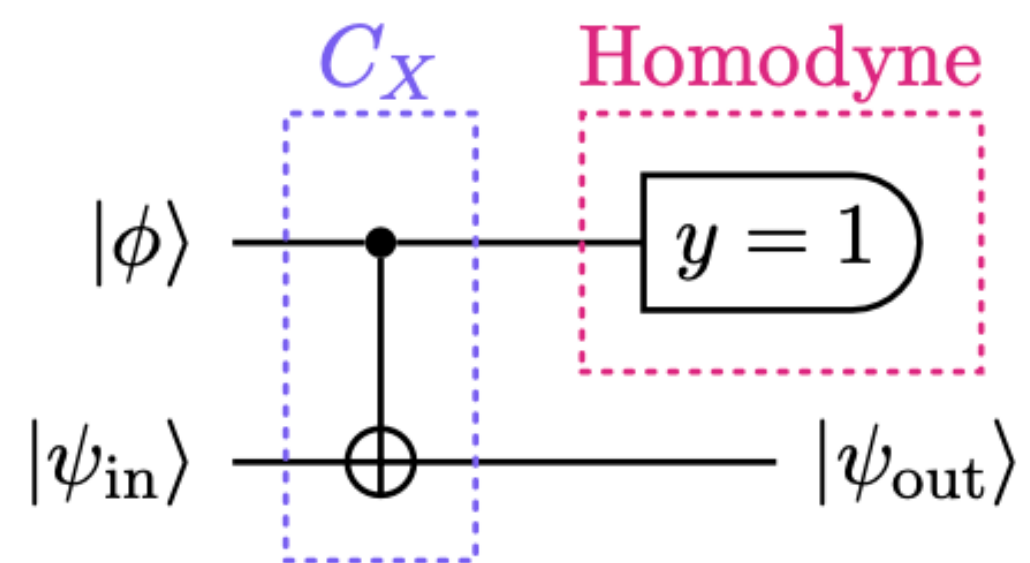
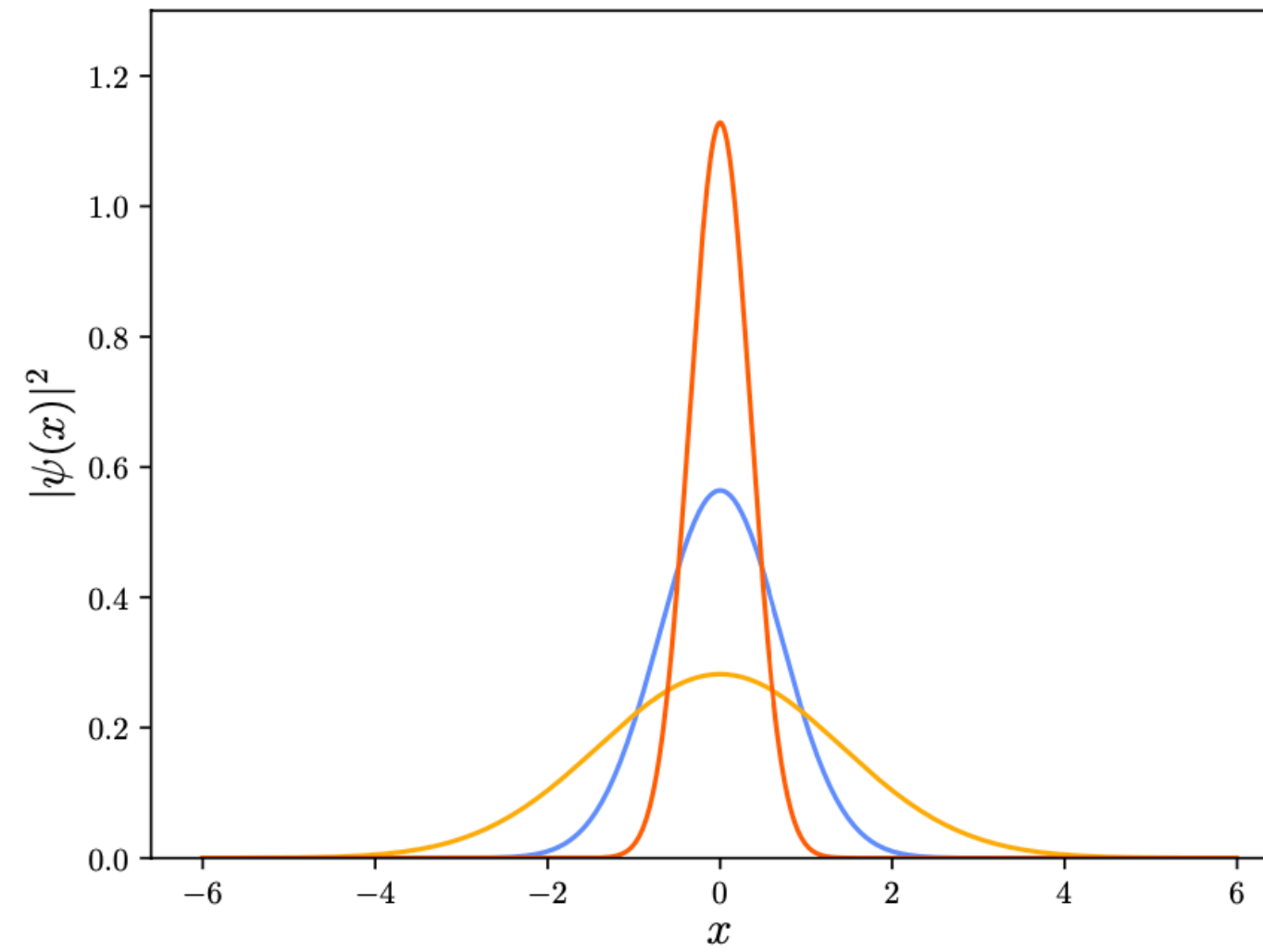
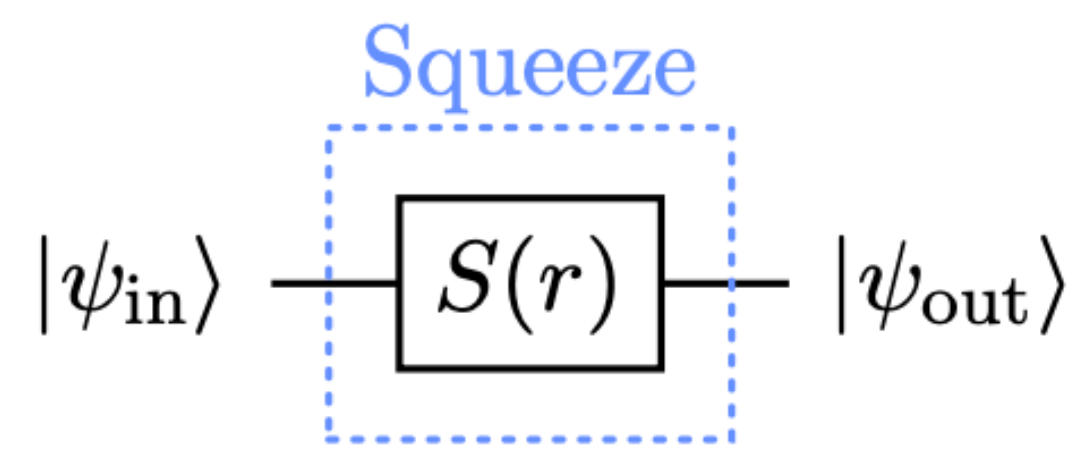
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$$\text{Controlled-X: } C_X(s; \hat{y}, \hat{p}_x) = e^{-is\hat{y}\hat{p}_x} \implies \hat{x} \rightarrow \hat{x} + s\hat{y}.$$

$$\text{Controlled-Z: } C_Z(s; \hat{x}, \hat{y}) = e^{is\hat{x}\hat{y}} \implies \text{momenta shifts instead}$$

Packages available to simulate CVQCs (e.g. StrawberryFields)

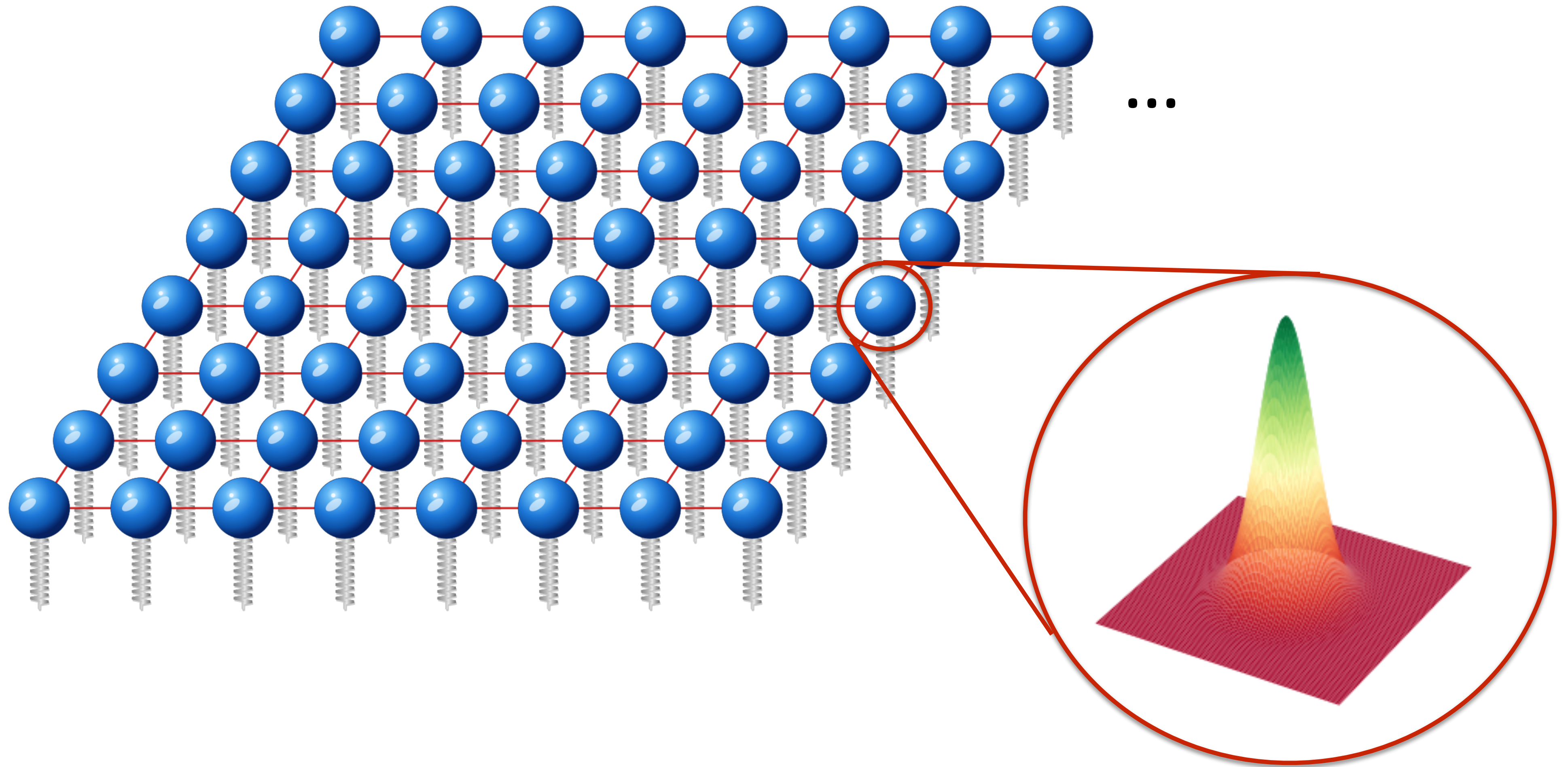
Let's try some simple examples



To build our QFT lattice we need the correct single qumode:

The tricky part before we can even couple them together

How do we put a potential on there?? $\equiv$ Hamiltonian evolution of QM



Evolving this single qumode is basically QM: we just need to be able to do the following to an arbitrary in-state ...

$$|\psi_{\text{out}}\rangle = e^{-i\mathcal{H}t}|\psi_{\text{in}}\rangle$$

We will use an ***evolver-state*** — a resource state that we ‘factor’ onto the in-state to make it evolve a small time step δt . It is an *ancilla* qumode with coordinate y , which looks like this:

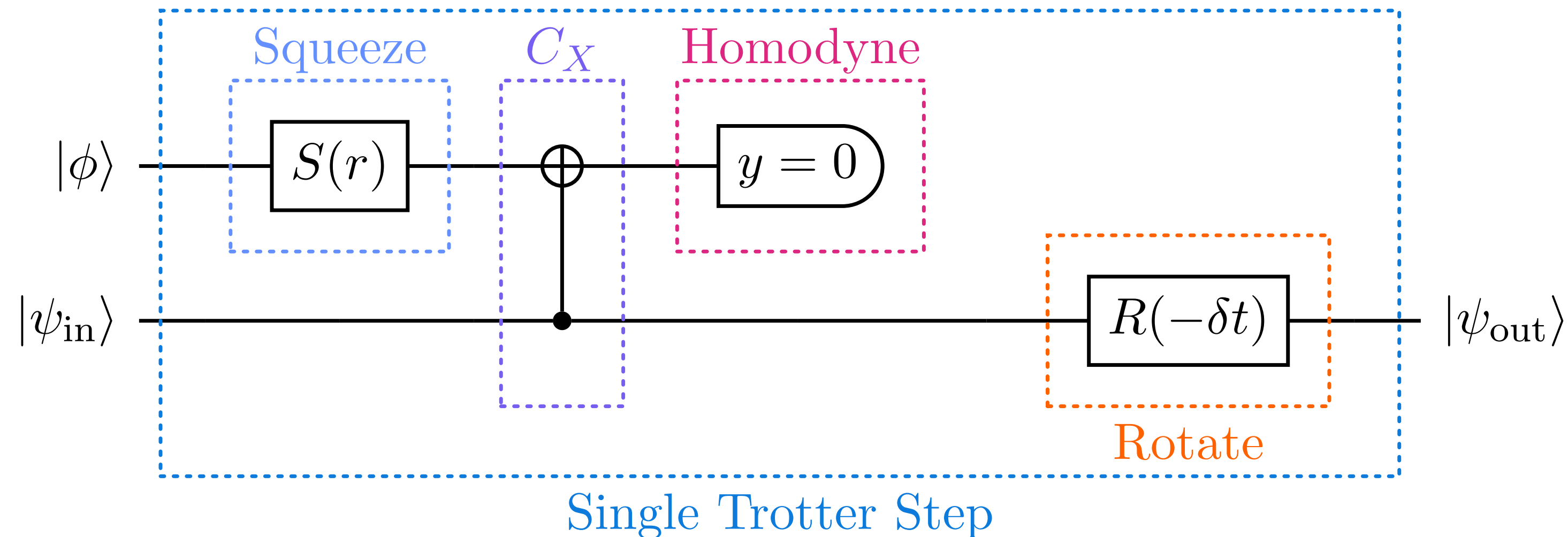
$$\langle y | \phi \rangle = \langle y | e^{-iH_1(\hat{y})\delta t} | \phi_0 \rangle$$

where $H_1(x)$ is the part of the Hamiltonian that is non-quadratic - (e.g. quartic potential), and $\langle y | \phi_0 \rangle$ is a wavefunction that looks like a top-hat (for reasons that will become clear).

Assuming we can make such a $\langle y | \phi \rangle$ state — by making a homodyne measurement we can use it to invoke Schrödinger evolution ...

SAA, Spannowsky, Williams

Evolver - “Gadget”:

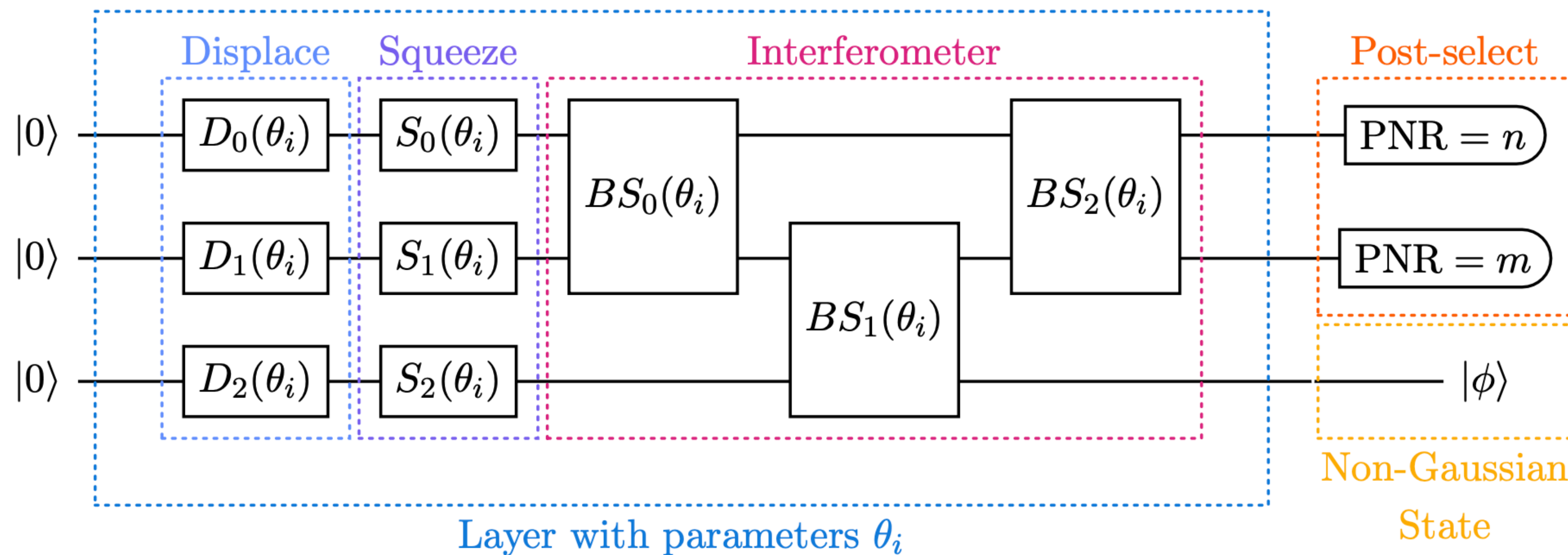


$$\langle x | \psi_{\text{out}} \rangle = \exp \left(-\frac{i}{2} (\hat{p}^2 + \hat{x}^2) \delta t \right) e^{-i H_1(\hat{x}) \delta t} \langle x | \phi_0 \rangle \langle x | \psi_{\text{in}} \rangle$$

“noise” function which is roughly constant if we choose $|\phi_0\rangle$ to be the top-hat

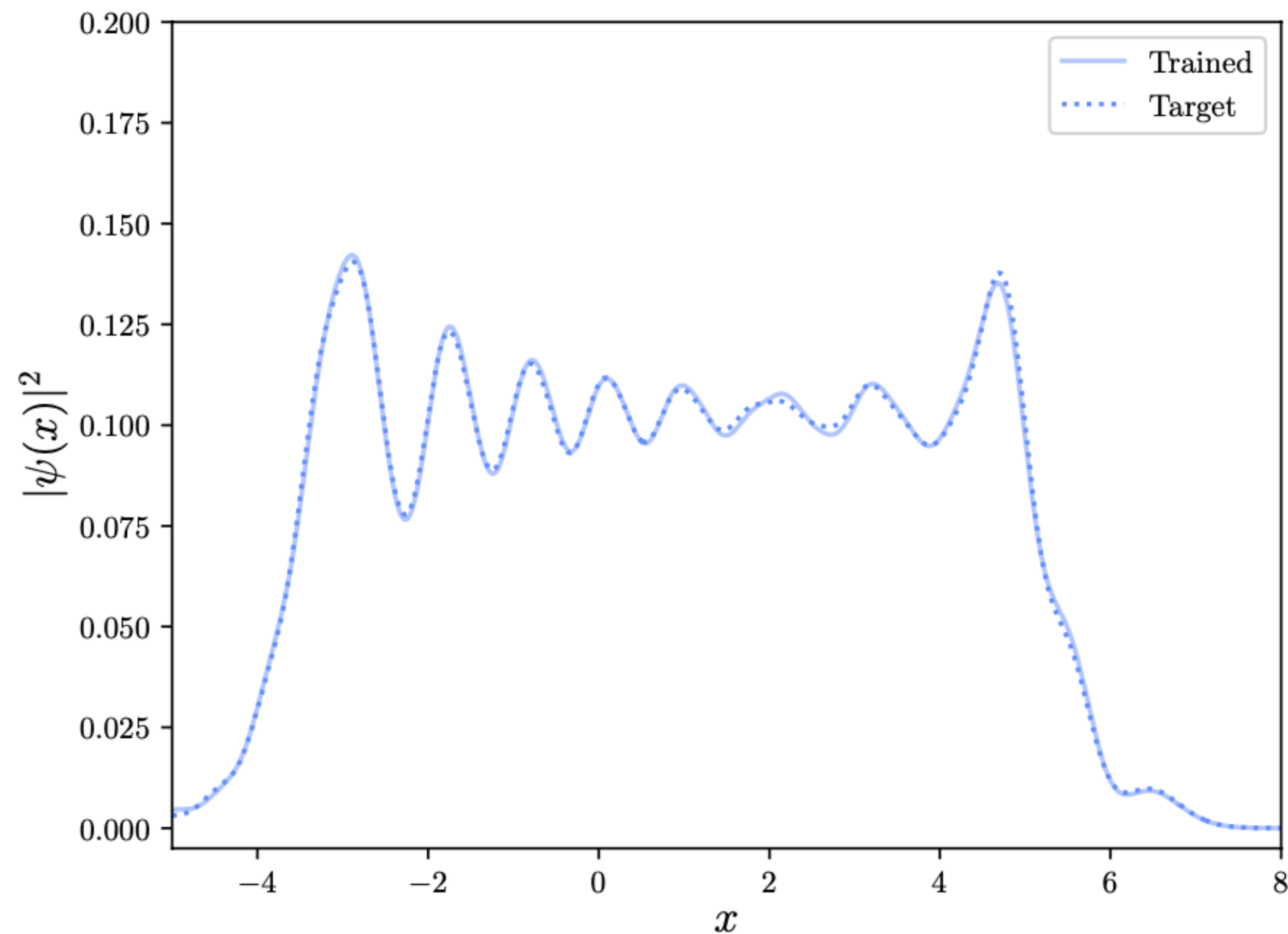
Machine learning the $\langle y | \phi \rangle$ evolver-state: [SAA, Spannowsky, Williams]

If we want to use an entirely photonic device we use photon measurement and ML to tune a circuit that gives desired non-Gaussian state on the ancilla qumode ... [Izaac, Myers, Sabapathy, Su, Weedbrook]



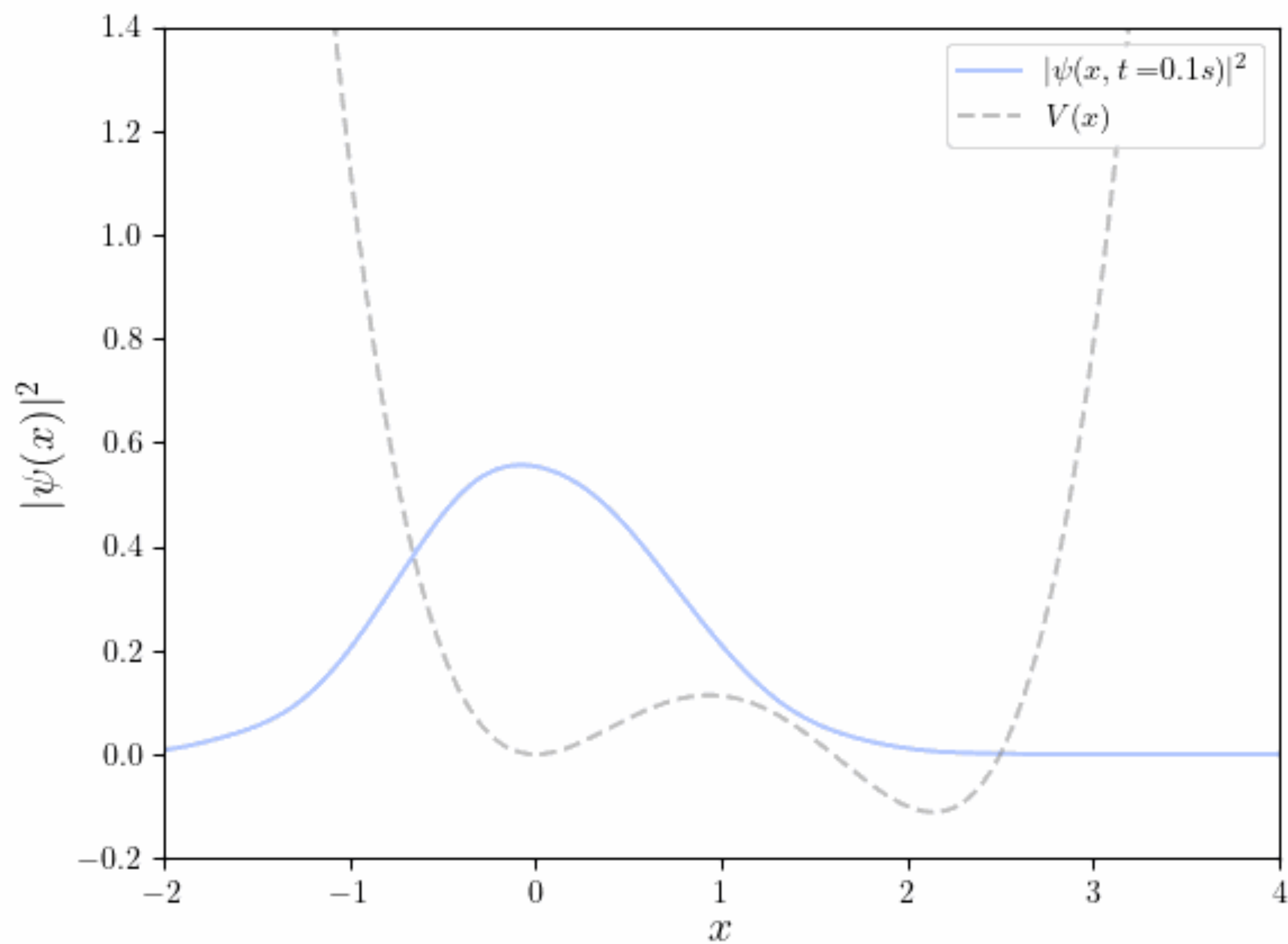
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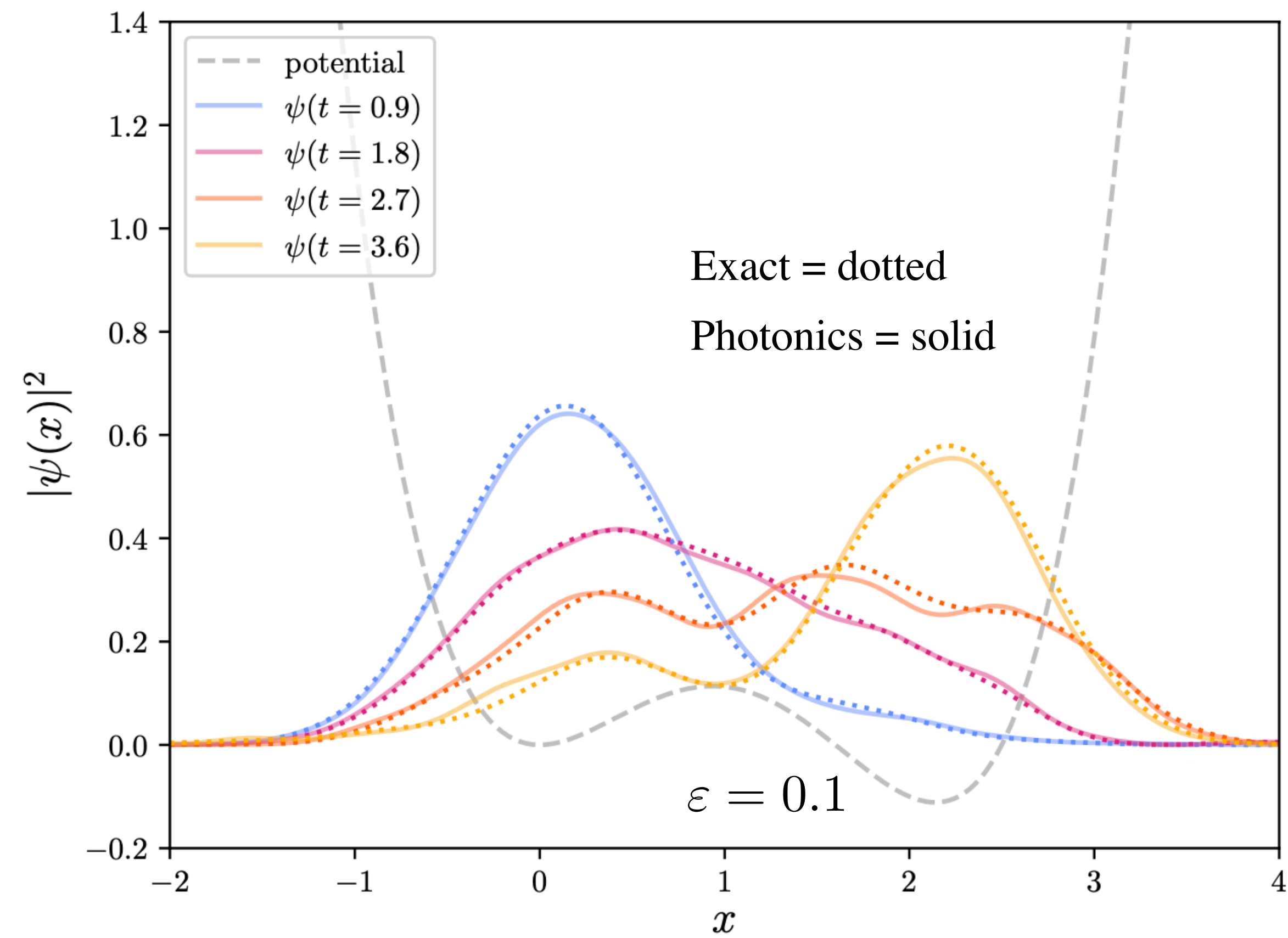


Evolving QM using an evolver state: example

$$V(x) = \frac{1}{8}(x^2 - 2x)^2 - \frac{\varepsilon}{8}x^3$$



Evolving QM using an evolver state: example



Building a QFT: The qumode lattice

Suppose we have a lattice of such qumodes at points $x_{n=1\dots N}$ with local quadrature variables $\hat{q}_n, \hat{p}_n$: then

$$\phi(x_n) = \hat{q}_n$$

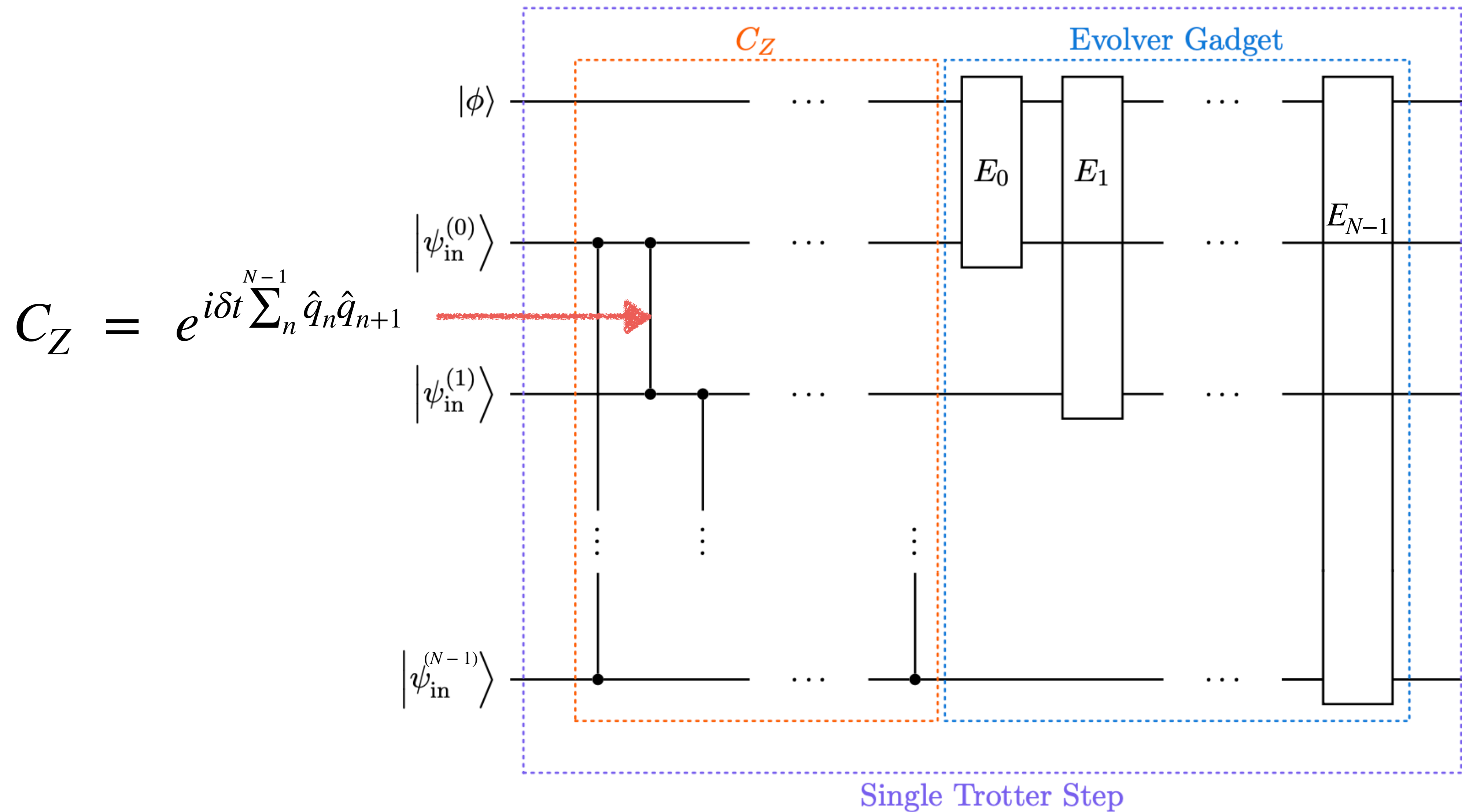
and we have the QFT Hamiltonian

$$H_{\text{QFT}} = \frac{a}{2} \sum_{n=1}^N \left[\hat{p}_n(t)^2 + \left(\frac{\hat{q}_{n+1}(t) - \hat{q}_n(t)}{a} \right)^2 + 2V(\hat{q}_n) \right]$$

upto the missing **Hopping terms** $H_{\text{QFT}} \supset -a^{-1} \sum_n \hat{q}_n q_{n+1}$

— these are the terms that couple our qumodes together

So circuit diagram for Trotter evolution QFT with N space-points looks like this:



4: Tensor Networks

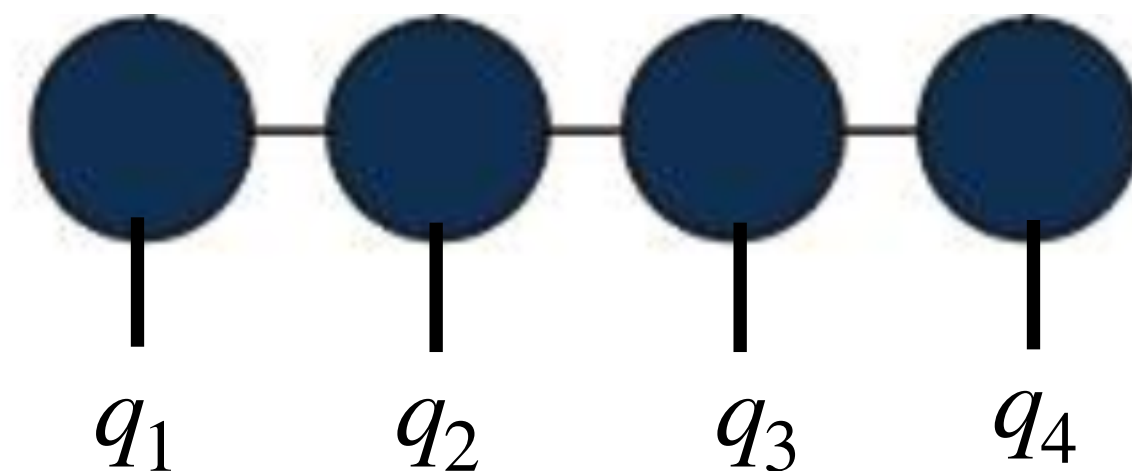
Tensor networks provide a classical method of Hamiltonian simulating this system with controlled entanglement

- Simulate local Hilbert space with discrete “qudits” with large dimensionality (say 20).
- Our 1-dimensional state is generally (with q_i ’s running over local quadrature values):

$$|\Psi\rangle = c^{q_1, \dots, q_N} |q_1, \dots, q_N\rangle$$

- Hamiltonian evolution use Time Evolving Block Decimation (TEBD) — way of systematically restricting entanglement growth as we keep applying $e^{-i\hat{H}\delta t}$ Trotter factors
- The trick of TEBD — rewrite each $c_{q_1 \dots q_N}$ as a **matrix product state (MPS)**

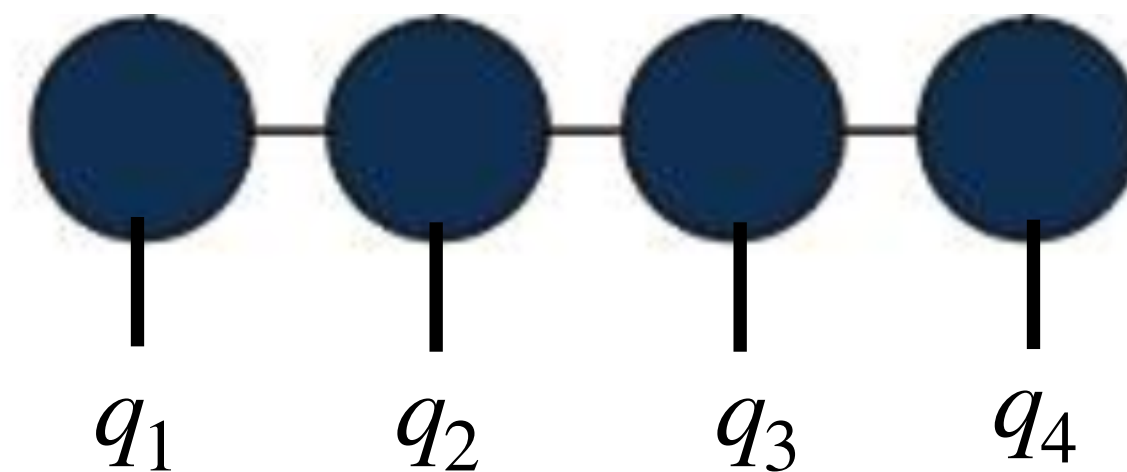
$$c^{q_1, \dots, q_N} = A_{\alpha_1}^{q_1} A_{\alpha_1, \alpha_2}^{q_2} \dots A_{\alpha_{N-1}, \alpha_N}^{q_{N-1}} A_{\alpha_N}^{q_N}$$



- At each bond can use single value decomposition to write the matrix as

$$c^{q_1, \dots, q_N} = \sum_i L^{q_1 \dots q_{m-1}, i} \lambda_i R^{i, q_{m+1} \dots q_N}$$

- This is the canonical Schmidt form — in which we say orthogonality centre is at node m
- By SVD we can move the orthogonality centre to any node
- The local λ_i Schmidt values can be truncated by maximum bond dimension and truncation value



Recall that given an initial state the Trotterisation is as follows:

$$|\Psi(t)\rangle = U(t) |\Psi(0)\rangle \quad \text{where} \quad U(t) = \left[\prod_i e^{-iH_i \delta t} \right]^{t/\delta t}$$

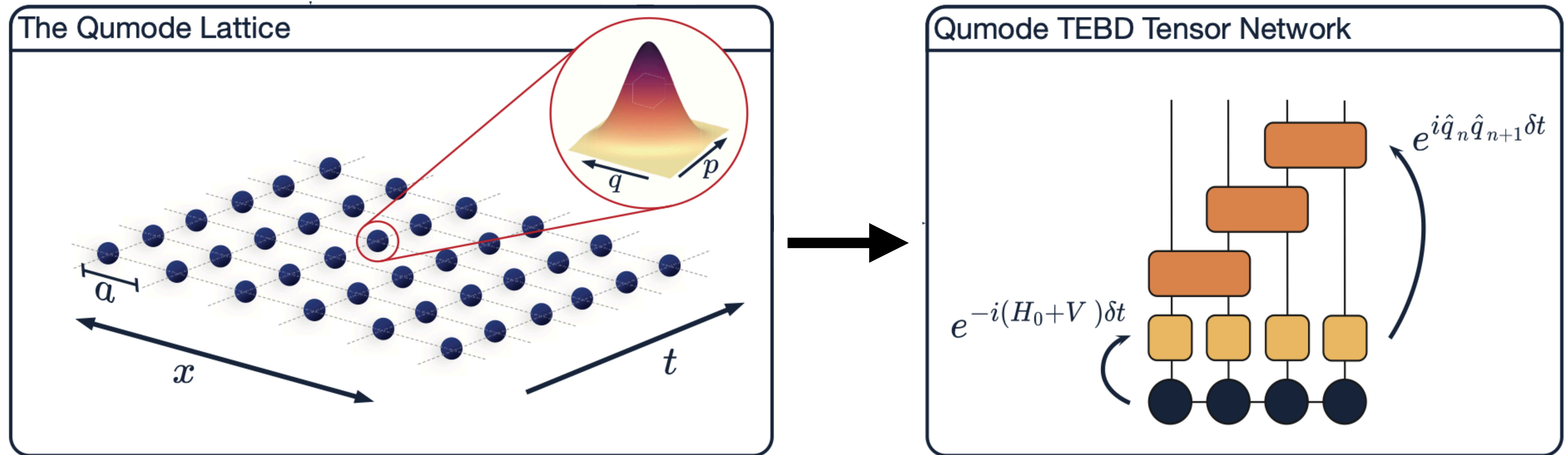
For example choose simple diagonal $\hat{q}$ basis with $q_n^j = -L + j \times \delta q$
(in practice use similar $\hat{q}$ diagonal DVR basis)

Then we can apply **onsite gates** locally on the nodes by simple matrix multiplication acting on the q_n indices, and **hopping gates** on adjacent pairs

$$\hat{q}_n = \text{diag}(q_n^1 \dots q_n^d) ,$$

$$\hat{p}_n^2 = -\frac{\hbar^2}{\delta q^2} \begin{pmatrix} 2 & -1 & & & \dots \\ -1 & 2 & -1 & & \dots \\ & -1 & 2 & & \dots \\ \vdots & & & \ddots & -1 \\ \vdots & & & -1 & 2 \end{pmatrix}$$

- Schematically ...

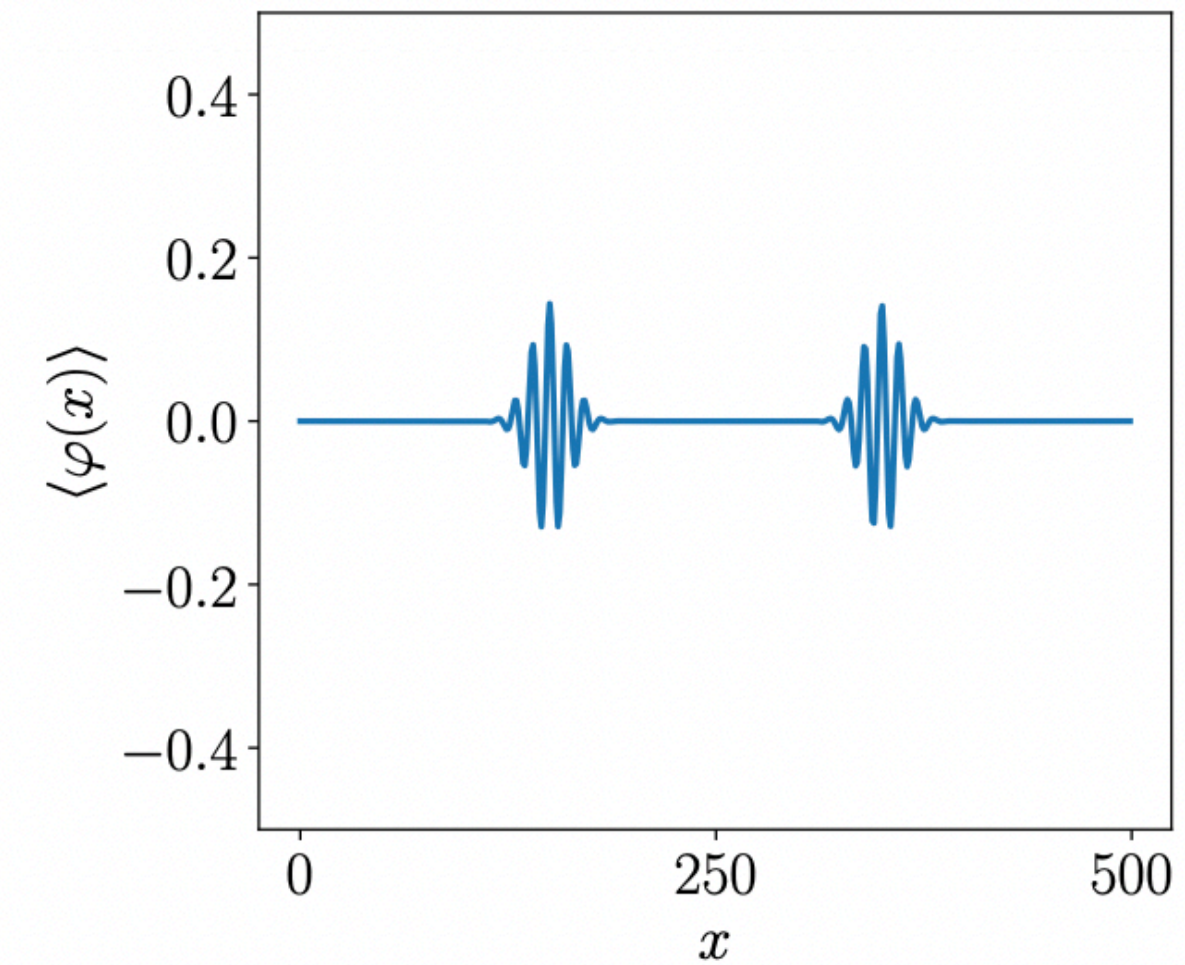


Scattering in $\lambda\phi^4$

Scattering in $\lambda\phi^4$ — initial state is two wave packets

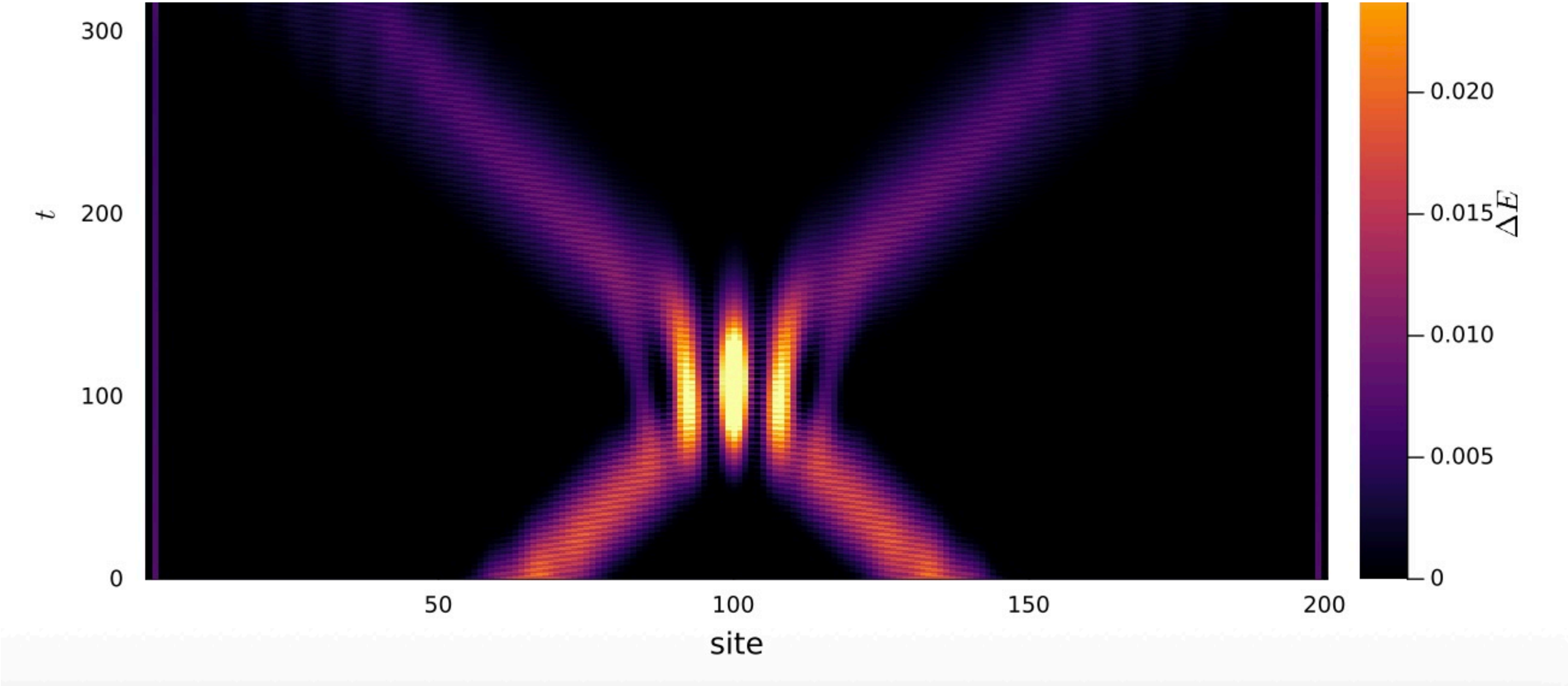
$$|\psi_n\rangle = \left(A^0 + \sum_{\alpha=0}^{N-1} \frac{1}{\sqrt{\mathcal{N}_\alpha}} \exp\left(-\frac{(k_\alpha - \bar{k})^2}{2\sigma^2}\right) \exp(ik_\alpha(x_n - \bar{x})) \hat{a}_n^\dagger \right) |0\rangle$$

and for now just take $|\Psi(0)\rangle = \prod_n |\psi_n\rangle$ as our initial wavefunction



Results using TEBD on GPUs

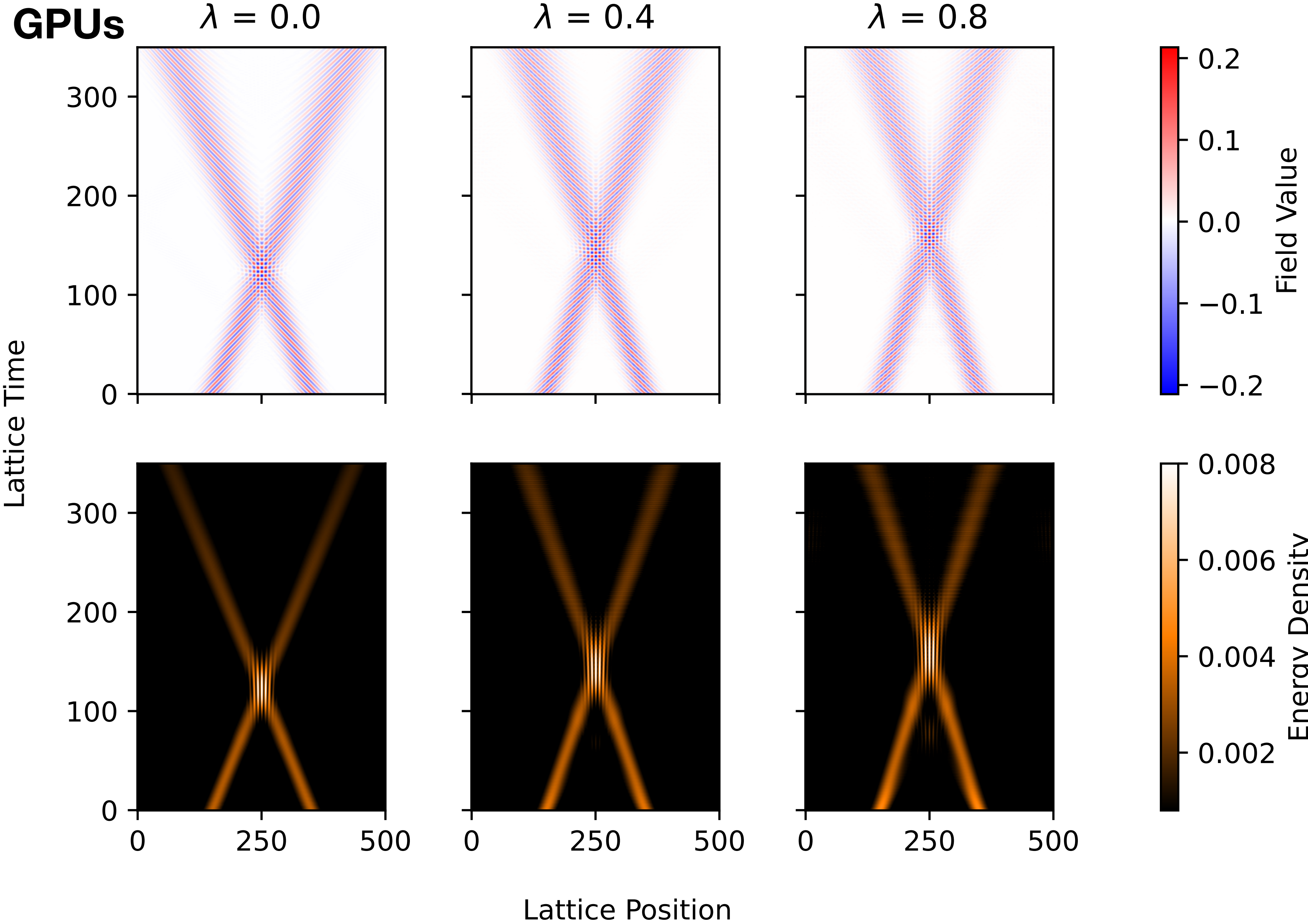
Wavepackets move
close to light speed
and are dispersive



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$\lambda\phi^4$ — is repulsive

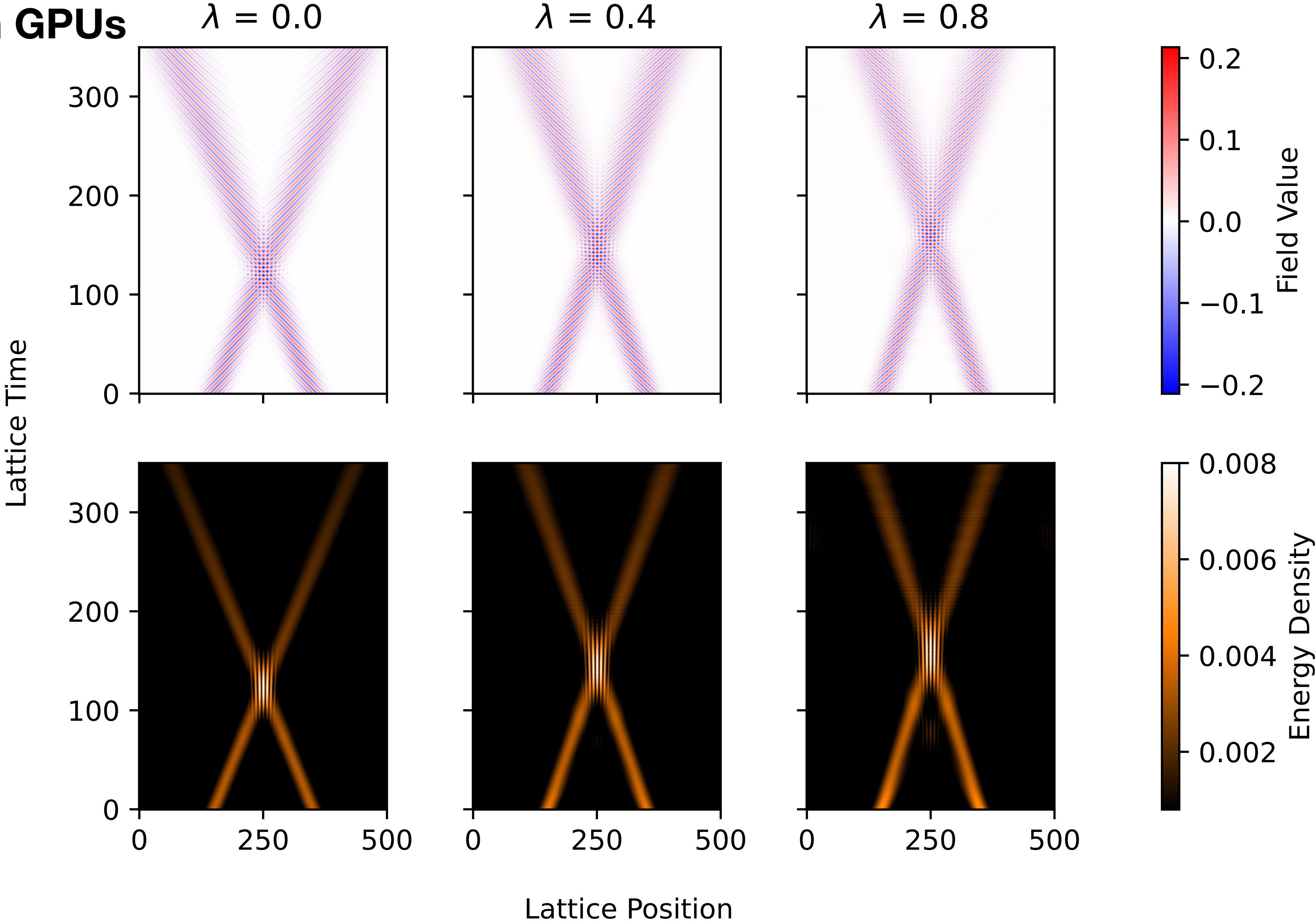


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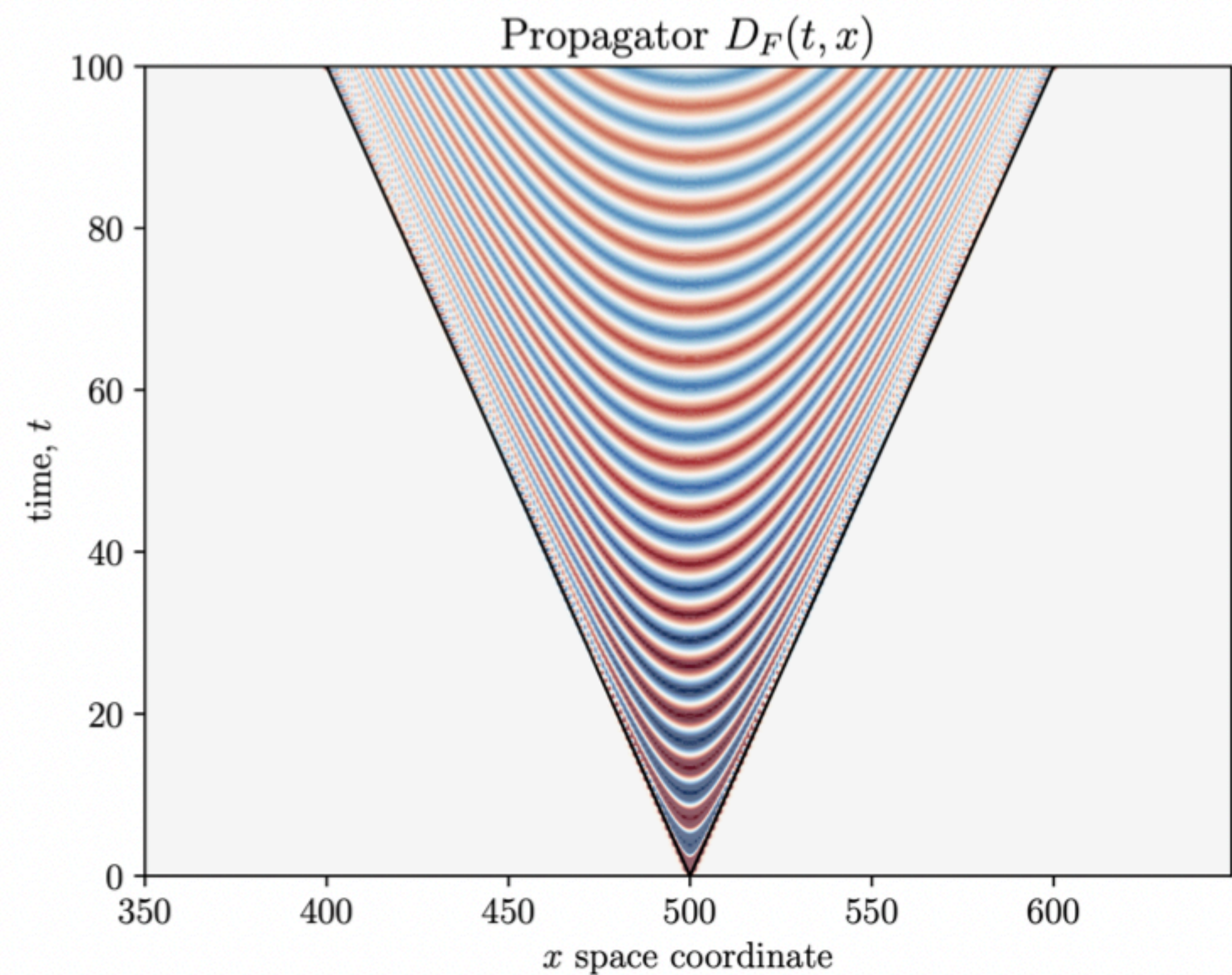
$\lambda\phi^4$ — is repulsive

“Your usage (0.301 kg CO2)
would enable you to boil
water for 509 cups of tea.”

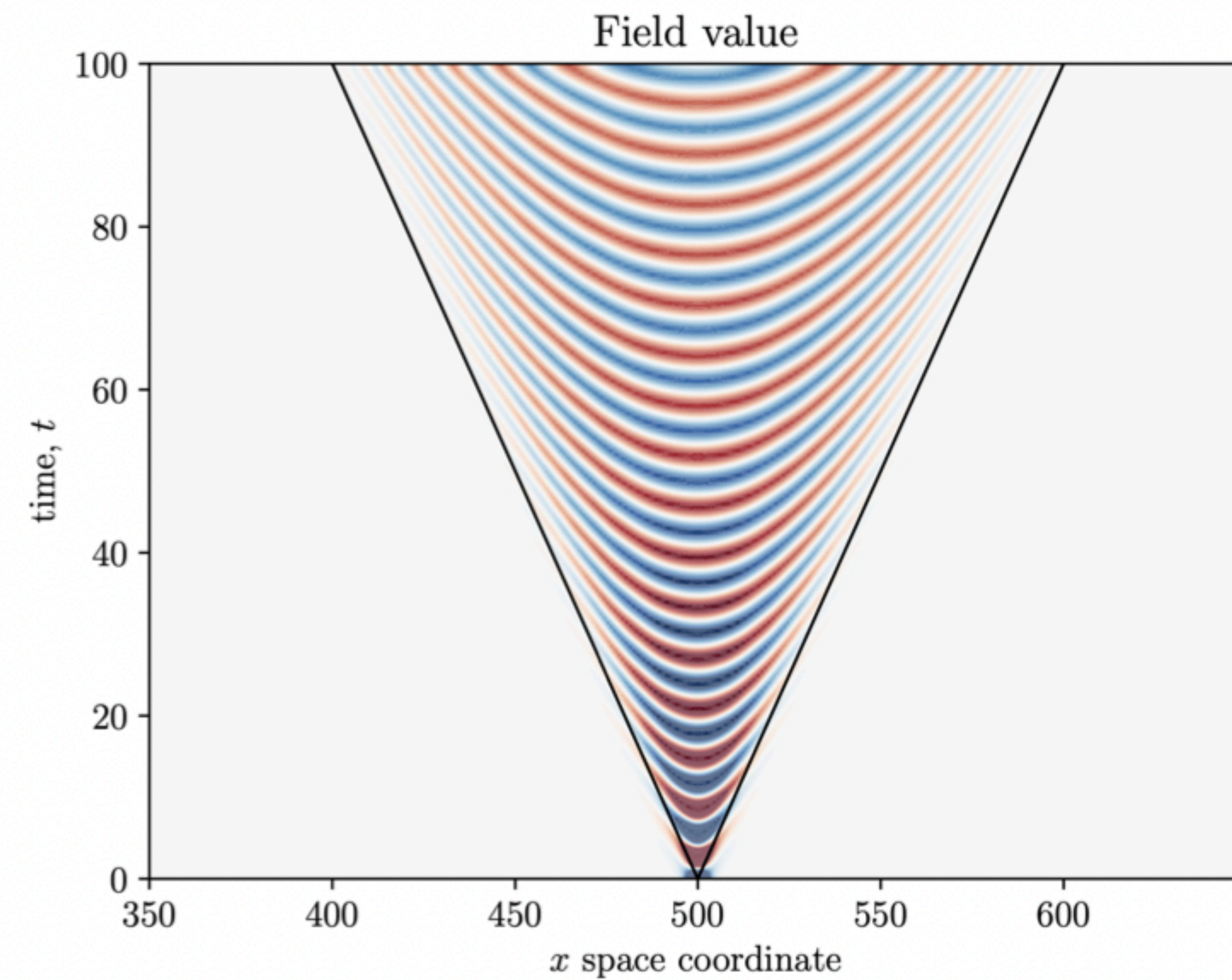


Results using TEBD on GPUs

Initial delta-function pulse: we see the correct two-point function



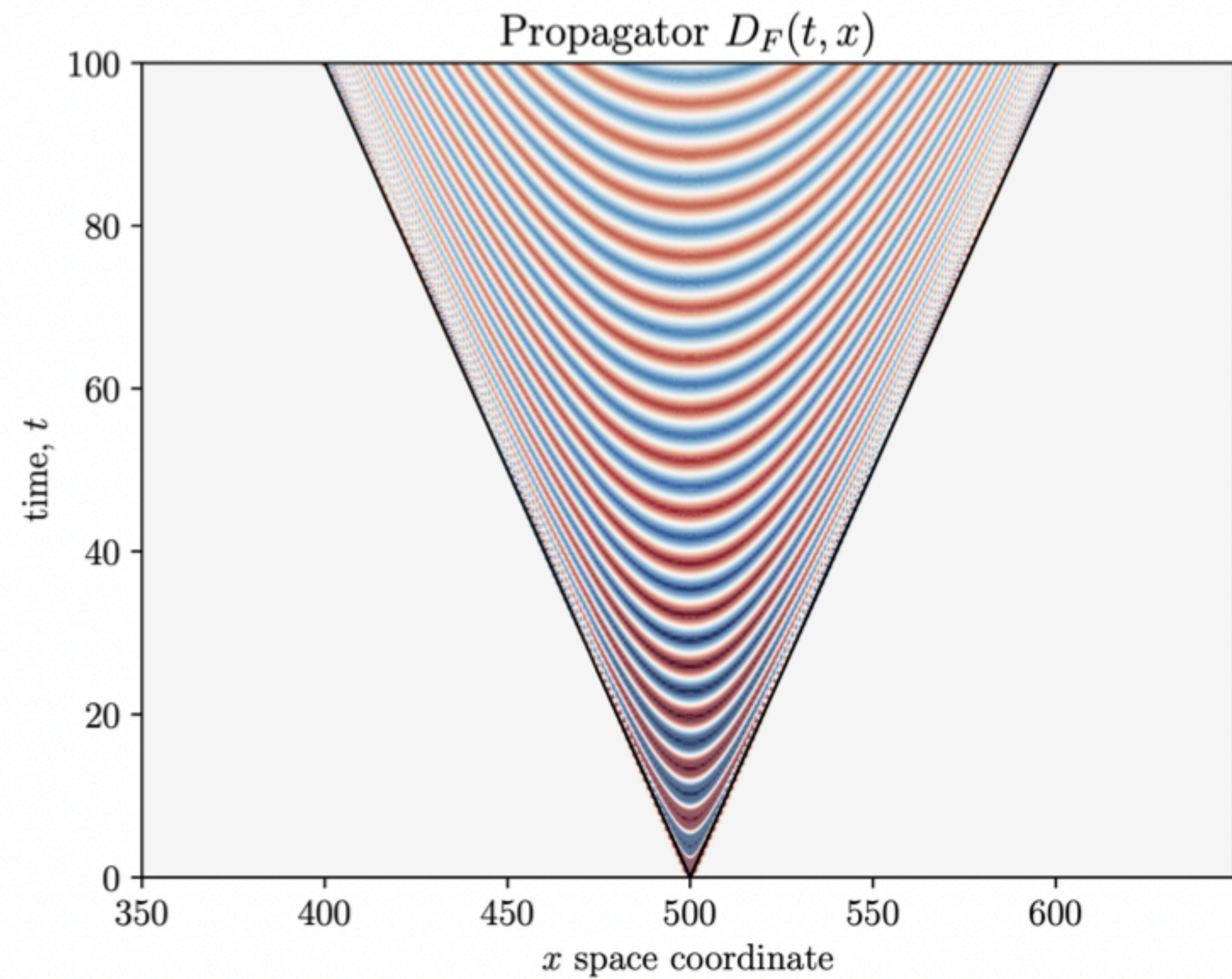
Theory



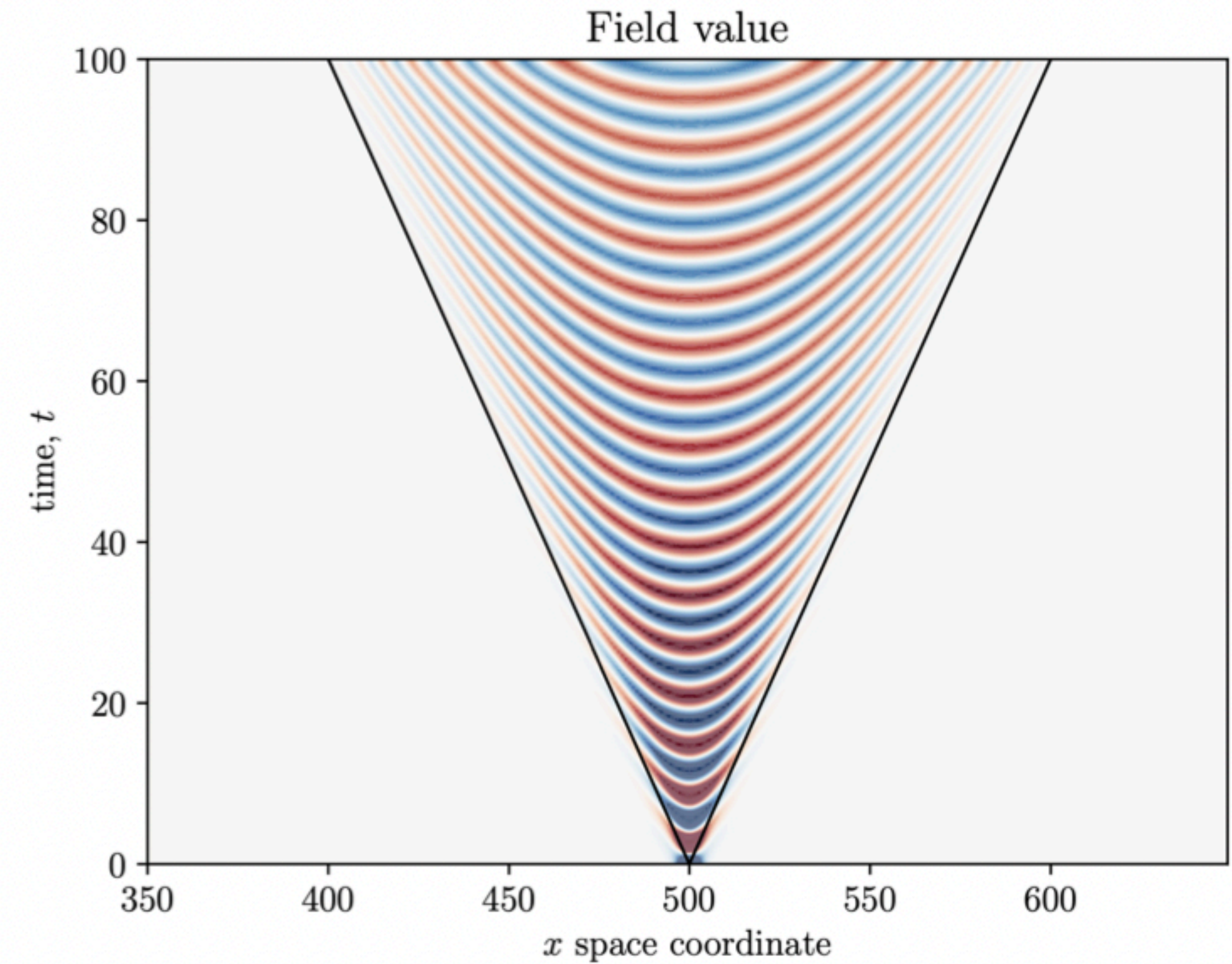
Simulation

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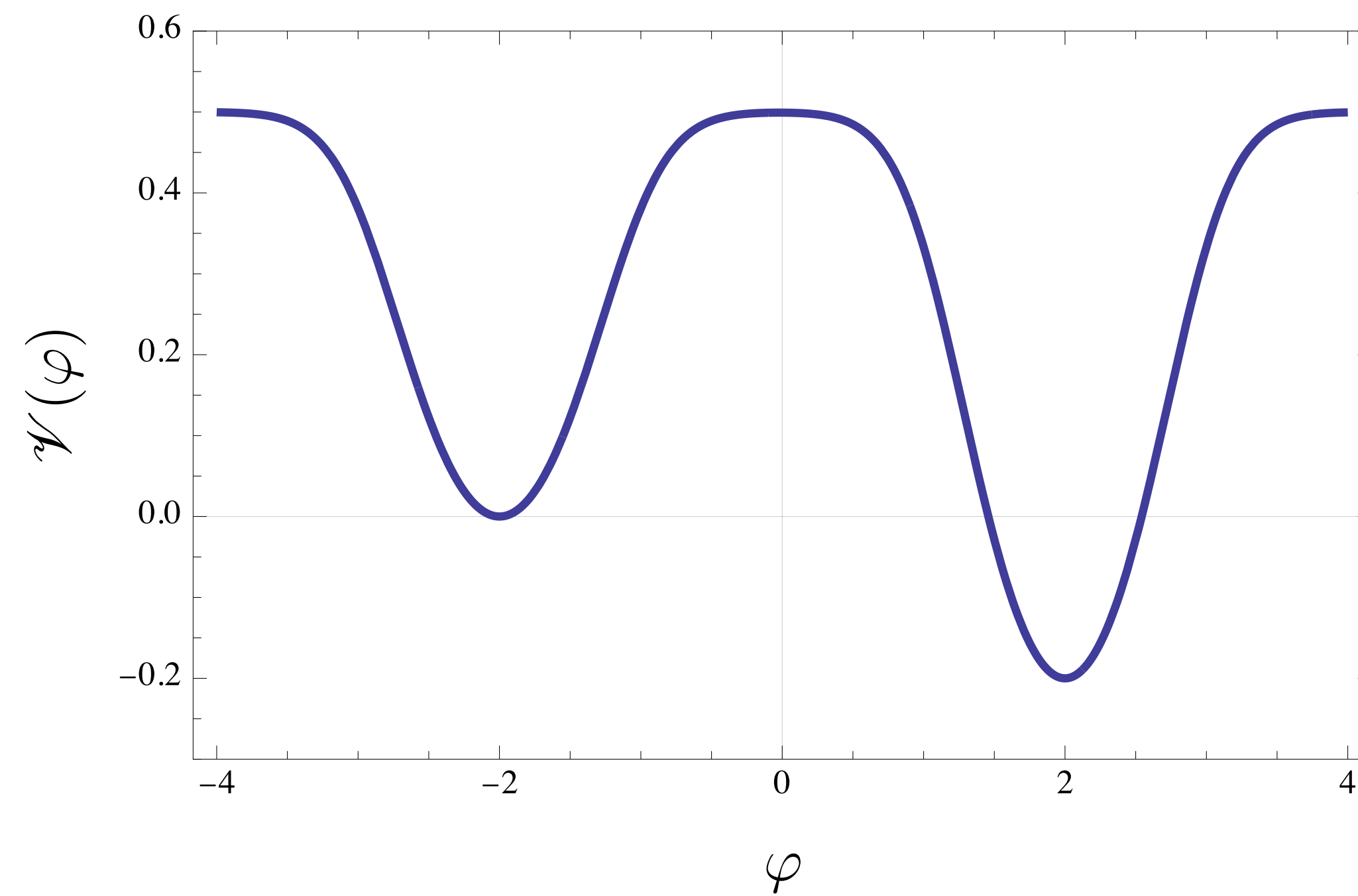
Simulation

Note this was all in Minkowski space but we could do all this in a gravitational background!!

Preparing the QFT vacuum and Phase transitions

Bubbles

Want to simulate bubble nucleation in a potential like ..

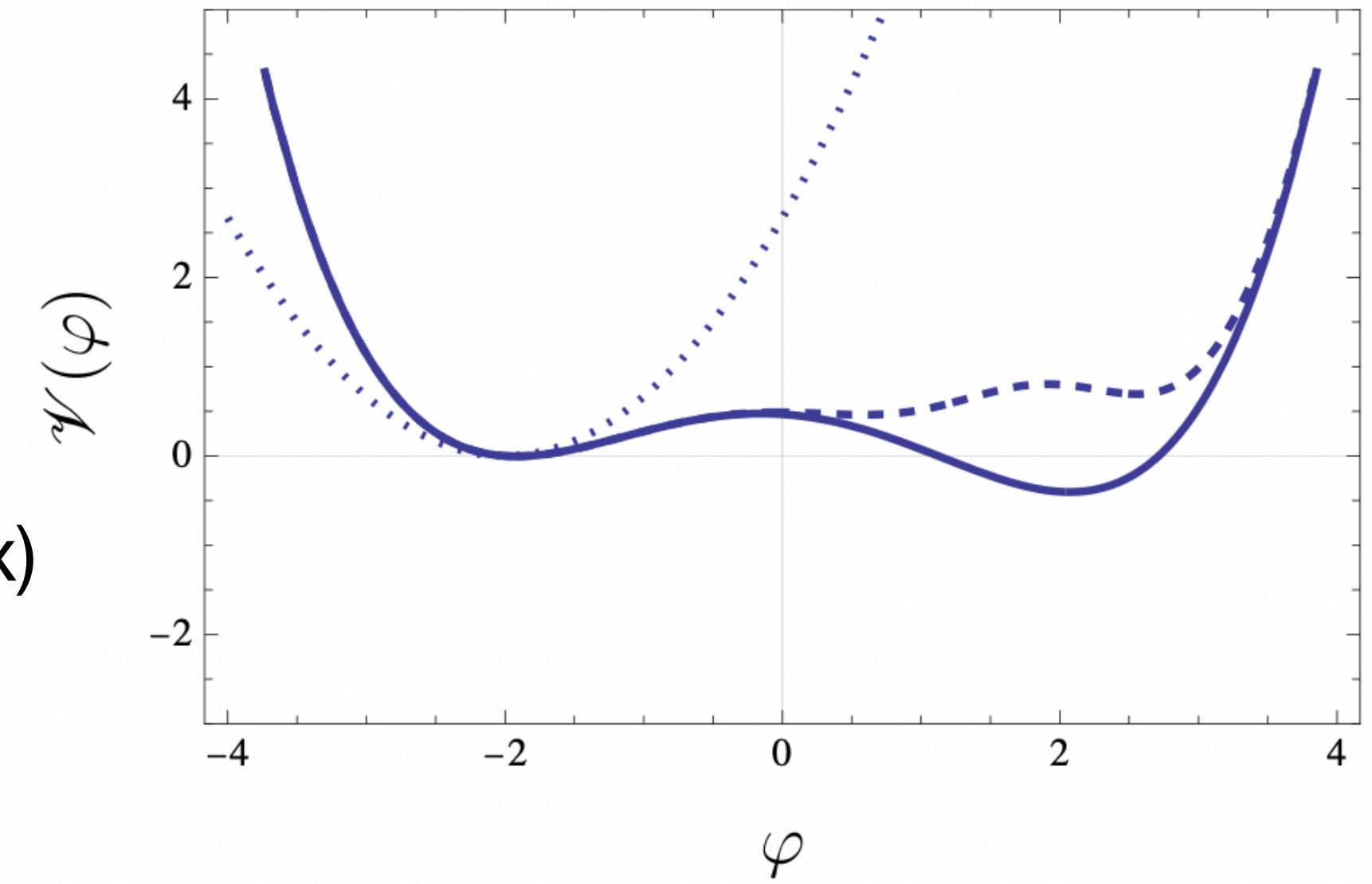


Prepare QFT groundstate using TEBD in imaginary time.

i.e. Do Trotter-Suzuki evolution through $-i\delta\tau$ steps and renormalise after each step, leaving lowest energy modes to dominate.

— Test result by measuring two-point correlations (covariance matrix)

$$C = \langle \hat{q} \hat{q}^T \rangle - \langle \hat{q} \rangle \langle \hat{q}^T \rangle$$



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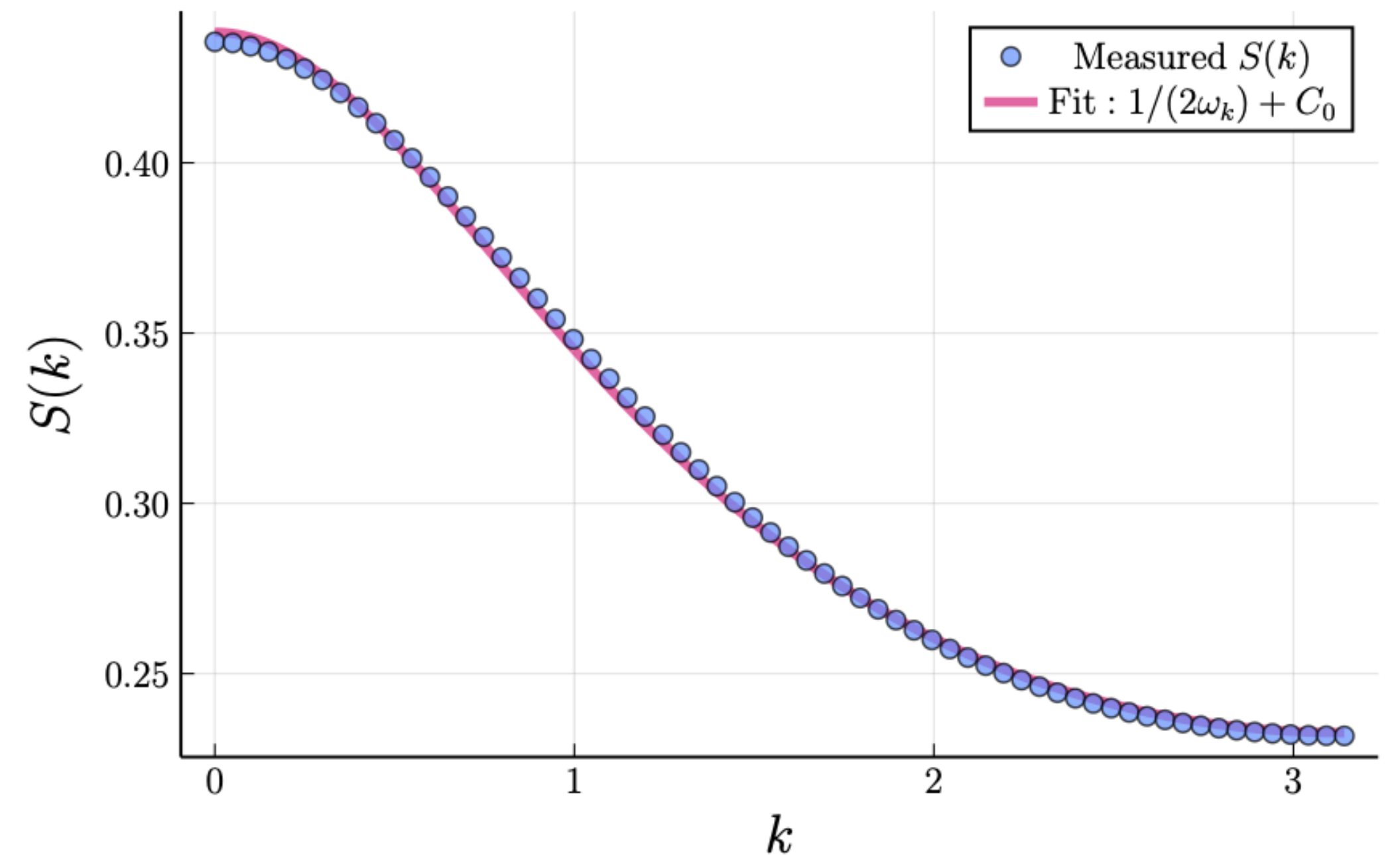
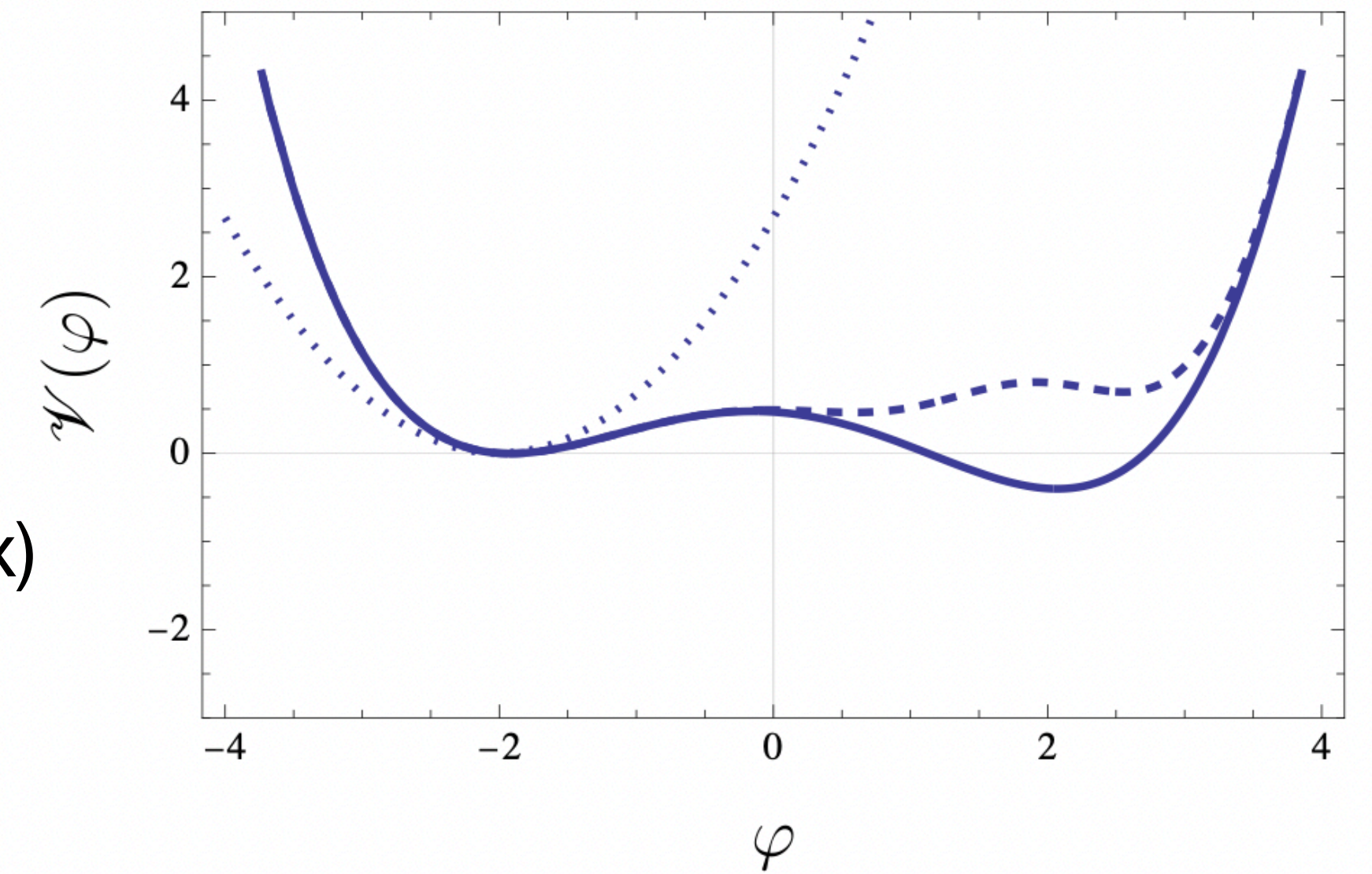
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$$C = \langle \hat{q} \hat{q}^T \rangle - \langle \hat{q} \rangle \langle \hat{q}^T \rangle$$

Then get $S(k)$ as its Fourier transform $S(k_\alpha) = 1/\omega_\alpha$

where

$$\omega_\alpha = \sqrt{m^2 + \frac{4}{a^2} \sin^2 \left(\frac{\pi\alpha}{N} \right)}$$



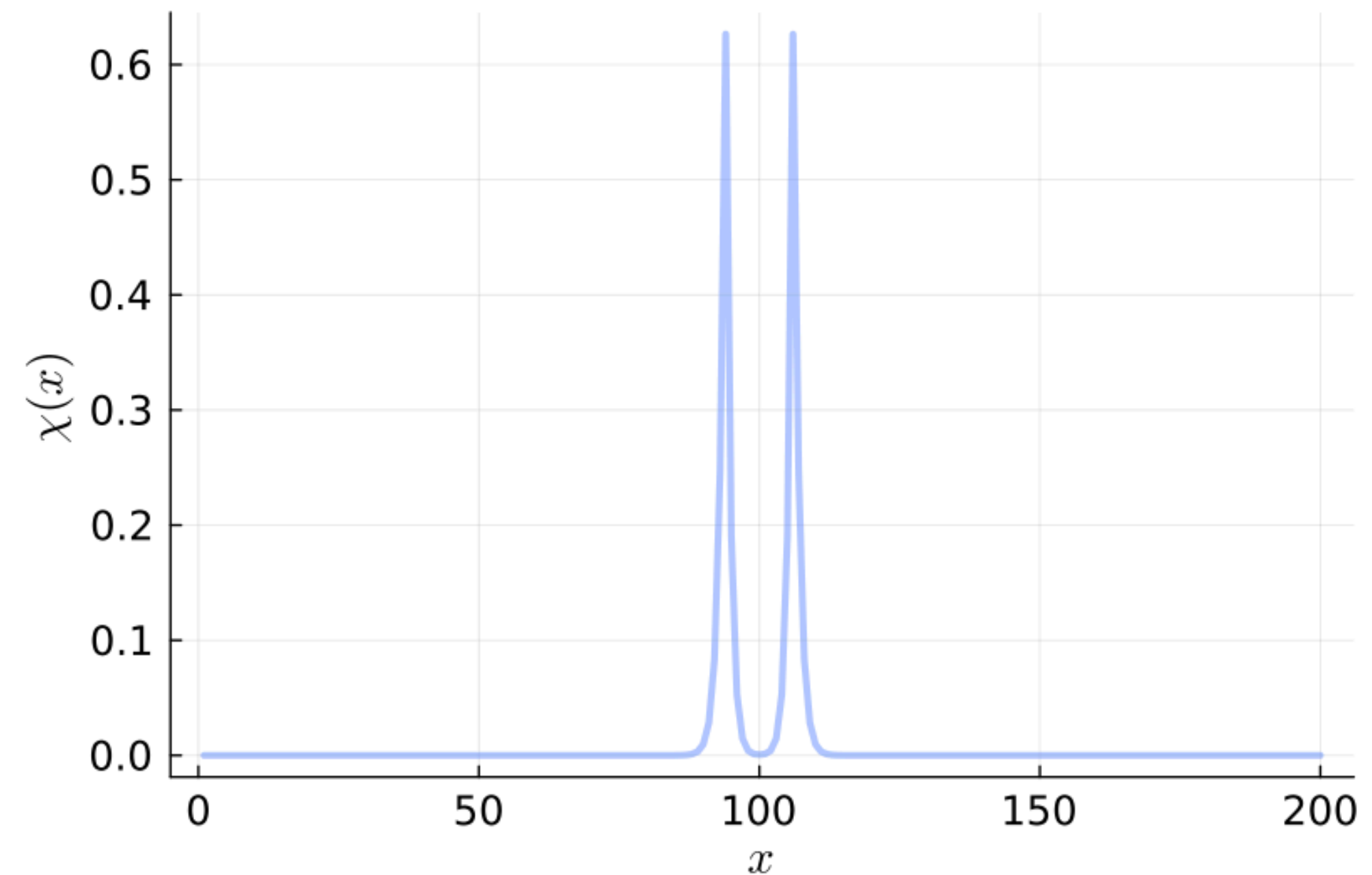
Genuine QFT vacuum decay — tunnelling itself extremely difficult in tensor networks because amplitude gets truncated (where the quantum rubber hits the road)

Linearise around known bubble solution: $\varphi(x, t) = \varphi_B(x, t) + \eta(x, t)$

$$\mathcal{L} = \frac{\eta_t^2}{2} - \frac{\eta_x^2}{2} - \frac{V''(\varphi_B(x))}{2} \eta^2$$

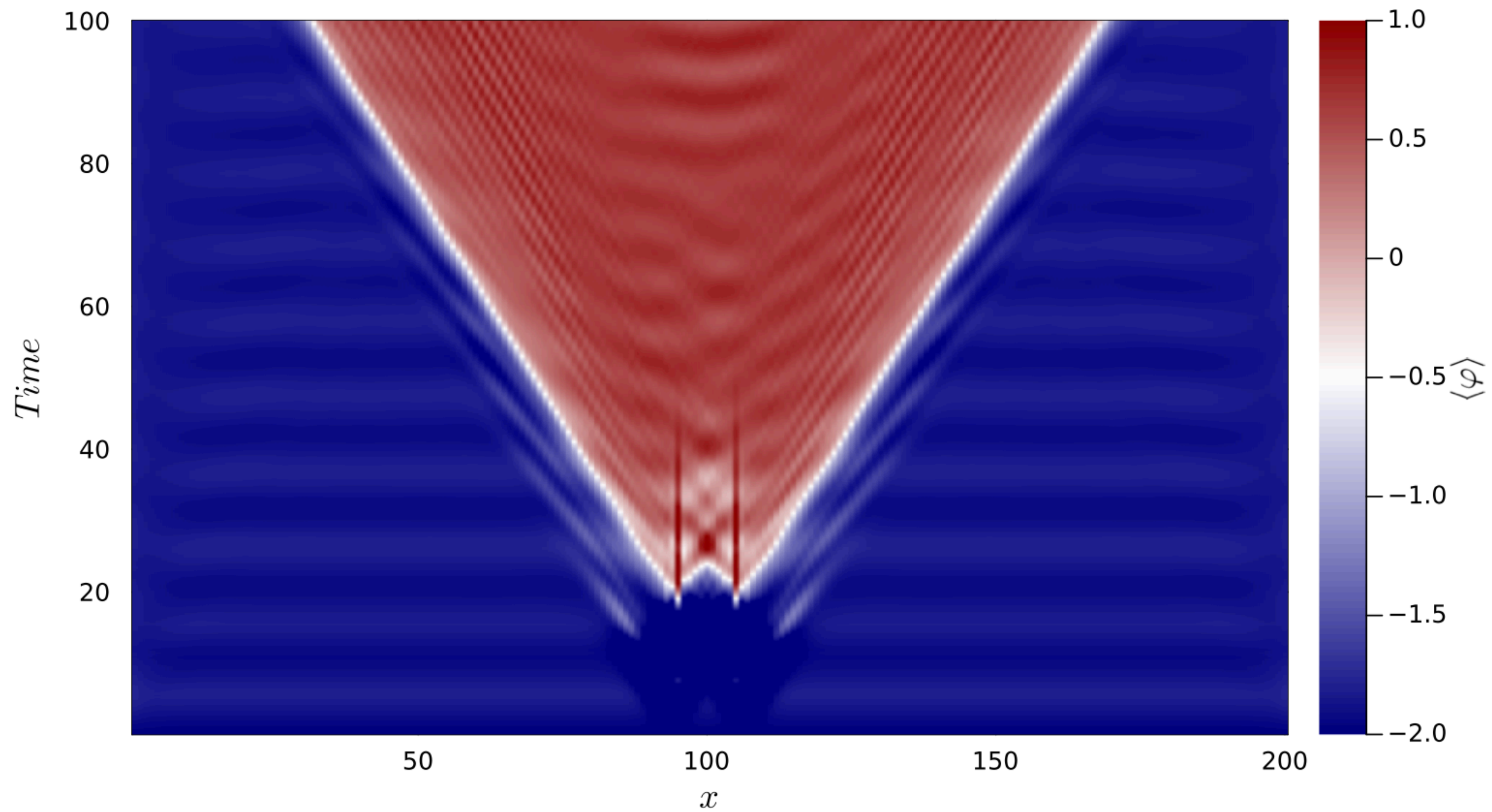
Separate variables $\eta = \chi(x)\tau(t)$ we have an exponentially growing mode $\tau(t) = Ae^{\omega_- t} + Be^{-\omega_- t}$ that obeys

$$-\chi_{xx} + (V''(\varphi_B(x)) + \omega_-^2)\chi = 0$$



Boundary conditions require $V''(\varphi_B) < 0$ so this is “under the barrier growth at the “bubble walls”: a small perturbation along the negative χ mode should grow

Genuine QFT vacuum decay



$$\varphi_{\text{fv}} = -2, \varphi_{\text{tv}} = 0.7, \varepsilon = 0.4$$

Summary

- Quantum computers have great potential to revolutionise many tasks
- The qumode lattice (either CVQC or large qudit approximation) is useful framework
- I showed you one type — linear lattice — many others (e.g. tree tensor networks, MERA lattice)
- Exciting playground for doing QFT.
- Ultimate aim: build continuous QFT to observe and measure tunnelling out of false vacuum
- Promises study of other non-perturbative processes in QFT: Unruh effect, Schwinger pair production, UV/IR mixed theories, Wormhole simulation, Black-Hole seeded phase transitions