From asymptotic symmetries to soft theorems and memory effects

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Introduction: Disclaimer and Motivation

Disclaimer: This talk is in my discomfort zone! I am not actively working on its topic!

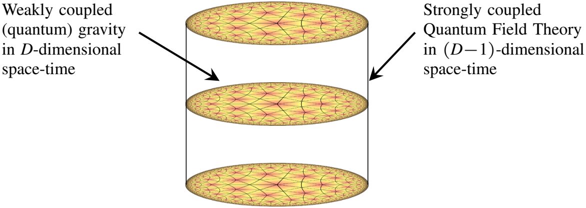
Main theme of the talk

Gauge theories (electromagnetism, gravity) in asymptotically flat spacetimes exhibit an infinite number of ***global*** symmetries/conserved charges that have interesting consequences.

- For gravity, this has been known since the '60s (Bondi, Van der Burg, Metzner, Sachs). Revisited in 2010 by Barnich and Troessaert. For other gauge theories, only recently realized. (Bieri, Garfinkle; Strominger et al.; Pasterski; Prabhu; Campiglia, Eyheralde, Laddha;...)
- These symmetries explain/link together diverse phenomena:
  1. soft theorems (Low; Weinberg)
  2. memory effects (Zel'dovich, Polnarev; Braginsky, Thorne; Christodoulou; Bieri, Garfinkle; Susskind; Pasterski)

Introduction: Disclaimer and Motivation

- Motivation: holography in asymptotically flat spacetimes?
  From



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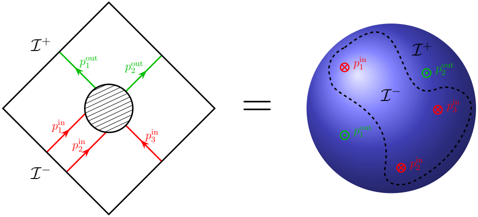


Figure: source: arXiv:1703.05448

Introduction: Disclaimer and Motivation

- The infinite number of symmetries in gravity in asymptotically flat spacetimes act as conformal transformations on the celestial sphere.

  $\Rightarrow$ Can one describe bulk gravitational scattering in terms of a conformal field theory on the celestial sphere?

- Other motivations:
  1. Contains some "textbook" lessons about gauge theories that dispel common folklore:

     > In the presence of boundaries, gauge symmetries can have physical effects!

  2. Combines themes of mathematical physics (holography, infinite-dimensional symmetries,...) with practical applications in Quantum Field Theory (soft theorems) and Classical Field Theory (memory effects)

     $\Rightarrow$ Good topic for a Supek colloquium!

Soft radiation in electromagnetism

- The main actor in this talk is soft radiation in gauge theories. Here, electrodynamics. In Quantum Electrodynamics: Bremsstrahlung.
- Accelerating charges emit radiation that travels at speed of light c.

$$\text{radiation field} \propto \frac{1}{r} \qquad \longleftrightarrow \qquad \text{Coulomb field} \propto \frac{1}{r^2}$$

  $\Rightarrow$ at large r effect of radiation dominates.
- Soft radiation: zero frequency/energy photons.
  Semi-classical analysis shows that there is an infinite number of soft photons emitted any time charged objects scatter.
- Deep IR phenomenon. Soft photon physics dictated by long-time behaviour of charges that emit them. Only depends on their initial and final velocities, not on details of scattering at intermediate times.
- Two examples of soft photon physics: soft photon theorem and electromagnetic memory effect.
- Rest of this talk: give short review of these 2 examples + explain how they are consequences of an infinite number of global symmetries.

Soft photon theorem

- Soft photons appear in the so-called "soft photon theorem". (Low; Weinberg)
- Consider scattering of n particles with momenta p_k^{in} and charges Q_k^{in} into m particles with momenta p_l^{out} and charges Q_l^{out} and S-matrix element

$$\langle \text{out} \,|\, S \,|\, \text{in} \rangle \,, \qquad |\text{in}\rangle = |p_1^{\text{in}}, \cdots, p_n^{\text{in}}\rangle \,, \quad |\text{out}\rangle = |p_1^{\text{out}}, \cdots, p_m^{\text{out}}\rangle$$

Then:

$$\langle \text{out} \,|\, a_+^{\text{out}}(q) S \,|\, \text{in} \rangle = e \underbrace{\left[\sum_{l=1}^{m} \frac{Q_l^{\text{out}} p_l^{\text{out}}.\epsilon^+}{p_l^{\text{out}}.q} - \sum_{k=1}^{n} \frac{Q_k^{\text{in}} p_k^{\text{in}}.\epsilon^+}{p_k^{\text{in}}.q} \right]}_{\text{soft factor, universal}} \langle \text{out} \,|\, S \,|\, \text{in} \rangle + \mathcal{O}(q^0) \,,$$

with $\langle \text{out} \,|\, a_+^{\text{out}}(q) S \,|\, \text{in} \rangle$ S-matrix element for emission of one photon with momentum q^μ and polarization ϵ_μ^+.

Soft photon theorem

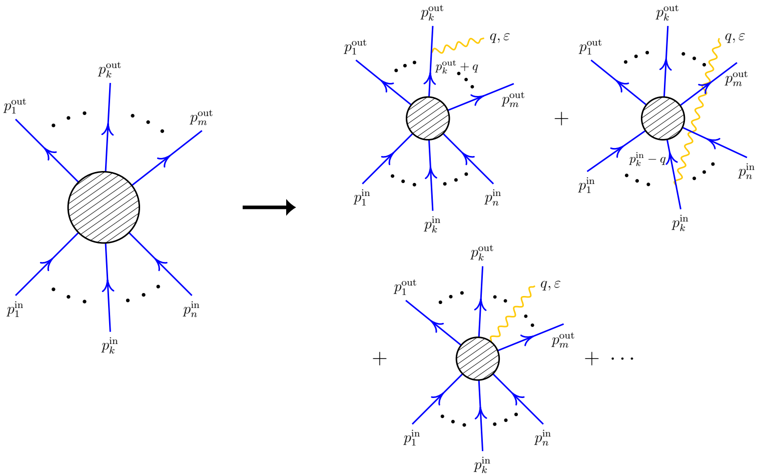
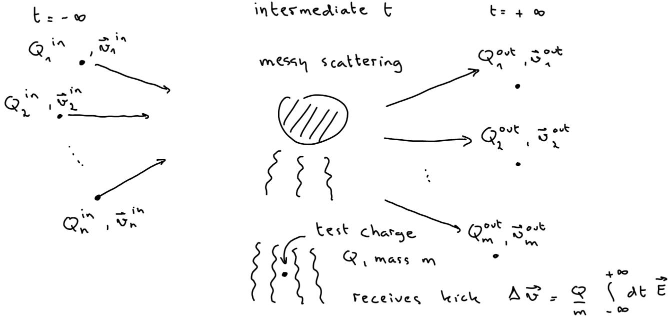


Figure: source: arXiv:1703.05448

Proof: evaluate Feynman diagrams on the right-hand side via LSZ reduction rule.

Memory effect

- Originally found in gravitational wave physics. (Zel'dovich, Polnarev; Braginsky, Thorne; Christodoulou)
- The passage of a gravitational wave through a configuration of test masses can cause a permanent displacement of the configuration.
- Electromagnetic analogue. (Bieri, Garfinkle; Susskind; Pasterski) Setup:



- Memory kick $\frac{Q}{m}\int_{-\infty}^{+\infty} \mathrm{d}t\, \vec{E}$ only depends on initial and final velocities $\vec{v}_i^{\,\text{in}}$, $\vec{v}_i^{\,\text{out}}$. Infrared, long-time phenomenon.

Asymptotic symmetries

- What do soft photons, the soft photon theorem and the memory effect have to do with symmetries? Which symmetries?
- Symmetries are linked to conservation laws via Noether's first theorem:

Noether's first theorem

There is a one-to-one correspondence between continuous symmetries of a physical system and on-shell conserved currents and charges.

- The theorem gives an expression for the conserved currents.
- Usually this is only discussed in the context of global symmetries. Electromagnetism

$$S = \int \mathrm{d}^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu \right) , \qquad \text{with } \partial_\mu j^\mu = 0 ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \qquad E_i = F_{ti} , \qquad B_i \propto \epsilon_{ijk} F_{jk} .$$

instead has a gauge symmetry

$$\delta A_\mu = \partial_\mu \lambda .$$

Asymptotic symmetries

- Let's be rebellious and apply Noether's first theorem anyhow... Noether's first theorem gives the following conserved currents:

$$J^\mu_\lambda = -F^{\mu\nu}\partial_\nu\lambda + \lambda j^\mu \,.$$

- This is at first sight an odd result:
  1. There's an uncountably infinite number of such currents, one for every $\lambda(x)$.
  2. It depends explicitly on the gauge parameter $\lambda(x)$. Physically meaningless?

- Maybe not... Note that on-shell, using the equation of motion $\partial_\mu F^{\mu\nu} = j^\nu$, we have

$$J^\mu_\lambda \approx -F^{\mu\nu}\partial_\nu\lambda + \lambda\partial_\nu F^{\nu\mu} = -\partial_\nu\left(\lambda F^{\mu\nu}\right)$$
$$\Rightarrow \qquad J^\mu_\lambda \approx \partial_\nu K^{\mu\nu}_\lambda \qquad \text{with} \quad K^{\mu\nu}_\lambda = -\lambda F^{\mu\nu} \,.$$

The charge is then

$$Q_\lambda = \int_V \mathrm{d}^3x\, J^0_\lambda \approx \int_V \mathrm{d}^3x\, \partial_i K^{0i}_\lambda = -\int_{\partial V} \mathrm{d}S_i \lambda F^{0i} = -\int_{\partial V} \mathrm{d}S_i \lambda E^i \,.$$

$Q_\lambda = 0$ for $\lambda \to 0$ at infinity. However, if $\lambda = 1$, $Q_1 =$ electric charge via Gauss' law...

Asymptotic symmetries

- There is physics hiding in these currents/charges. How to make sense of them?
- In the presence of boundaries, gauge symmetries become a bit more subtle.

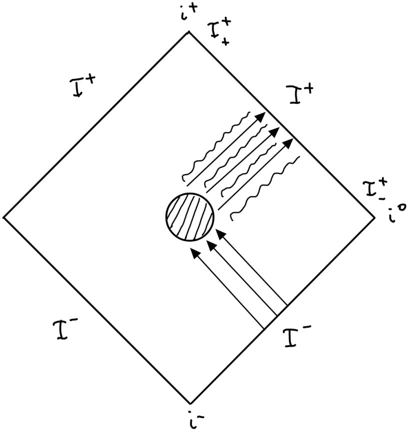
$$\text{theory} \;=\; \text{action} \;+\; \text{boundary conditions for the fields}$$

  Theories with the same action are different if the boundary conditions are different!
- Symmetries of a theory preserve both the action, as well as the boundary conditions! Only gauge symmetries that preserve the boundary conditions remain as symmetries!
- Of 2 kinds:
  1. Leave boundary conditions trivially invariant, i.e., parameter $\lambda \to 0$ towards boundary. Trivial or proper gauge transformations with $Q_\lambda = 0$.
  2. Leave boundary condition invariant in non-trivial way. $Q_\lambda \neq 0$. Should be viewed as global symmetries.

$$\text{Asymptotic symmetries} \;=\; \frac{\text{gauge symmetries preserving boundary conditions}}{\text{trivial gauge transformations}}$$

Asymptotic symmetries

- Let's consider the asymptotic symmetries of a radiative scattering problem in electromagnetism. For simplicity, scattering of massless charged particles.
- Penrose diagram:



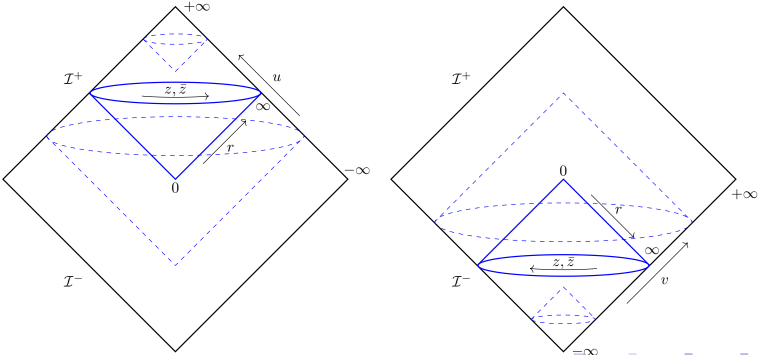
Asymptotic symmetries

- Boundary for radiative scattering in flat spacetime = past and future null infinity $\mathcal{I}^{\pm} = \mathbb{R} \times S^2$.
- Minkowski spacetime in retarded spherical coordinates $(u, r, z, \bar{z})$ $(u = t - r)$:

$$ds^2 = -du^2 - 2du dr + 2r^2 \gamma_{z\bar{z}} dz d\bar{z}\,, \qquad \mathcal{I}^+ = u \text{ constant},\ r \to \infty\,.$$

In advanced spherical coordinates $(v, r, z, \bar{z})$ $(v = t + r)$:

$$ds^2 = -dv^2 + 2dv dr + 2r^2 \gamma_{z\bar{z}} dz d\bar{z}\,, \qquad \mathcal{I}^- = v \text{ constant},\ r \to \infty\,.$$



Asymptotic symmetries

- Which boundary conditions for the field A_μ to impose? Part science, part art.

  E.g., at $\mathcal{I}^+$ (analogous for $\mathcal{I}^-$):

  1. Fall-offs for A_μ: (with $z^A = (z, \bar{z})$)

  $$A_u = \frac{A_u^{(1)}(u, z, \bar{z})}{r} + \mathcal{O}(r^{-2})\,, \qquad A_r = \frac{A_r^{(1)}(u, z, \bar{z})}{r} + \mathcal{O}(r^{-2})\,,$$
  $$A_A = A_A^{(0)}(u, z, \bar{z}) + \mathcal{O}(r^{-1})\,.$$

  2. Finite energy condition. A_μ becomes pure gauge (vacuum) for $u = \pm\infty$.

  $$A_A^{(0)}(u = \pm\infty) = \partial_A \phi_\pm \qquad \text{with} \qquad \phi_\pm = \phi_\pm(z^A)\,.$$

  3. Antipodal matching at spatial infinity:

  $$F_{\mu\nu}(-t, -\vec{r}) = -F_{\mu\nu}(t, \vec{r})\,.$$

Asymptotic symmetries

- Expanding the gauge parameter near $\mathcal{I}^{\pm}$ as

$$\mathcal{I}^{+} \ : \ \lambda = \lambda^{+}(u,z,\bar{z}) + \mathcal{O}(r^{-1})\,, \qquad \mathcal{I}^{-} \ : \ \lambda = \lambda^{-}(v,z,\bar{z}) + \mathcal{O}(r^{-1})\,,$$

one sees that $\delta A_u = \partial_u \lambda$ preserves the boundary condition $A_u = \mathcal{O}(1/r)$, provided

$$\lambda^{+} = \lambda^{+}(z,\bar{z})\,, \qquad \lambda^{-} = \lambda^{-}(z,\bar{z})\,.$$

- Charge conservation also requires an antipodal matching for λ:

$$\lambda^{+}(z^A) = \lambda^{-}(-z^A)\,.$$

Summarizing, asymptotic symmetries are “gauge transformations” with parameters λ that are independent of u and v at null infinity and antipodally matched.

Asymptotic symmetries

- We can then evaluate the "soft charges" Q_λ at $\mathcal{I}^\pm$ for these boundary conditions and parameters λ. They consist of two parts:

$$Q_\lambda^+ = \underbrace{\int_{\mathcal{I}^+} \mathrm{d}u\, \mathrm{d}^2z\, \sqrt{\gamma}\gamma^{AB} F_{uA}\partial_B\lambda^+}_{\text{soft part: } Q_\lambda^{+S}} + \underbrace{\int_{\mathcal{I}^+} \mathrm{d}u\, \mathrm{d}^2z\, \sqrt{\gamma} r^2 \lambda^+ j_u}_{\text{hard part: } Q_\lambda^{+H}} ,$$

$$Q_\lambda^- = \underbrace{\int_{\mathcal{I}^-} \mathrm{d}v\, \mathrm{d}^2z\, \sqrt{\gamma}\gamma^{AB} F_{uA}\partial_B\lambda^-}_{\text{soft part: } Q_\lambda^{-S}} + \underbrace{\int_{\mathcal{I}^-} \mathrm{d}v\, \mathrm{d}^2z\, \sqrt{\gamma} r^2 \lambda^- j_v}_{\text{hard part: } Q_\lambda^{-H}} ,$$

- They are both well-defined, i.e., finite. They are also conserved due to antipodal matching:

$$Q_\lambda^{+S} + Q_\lambda^{+H} = Q_\lambda^{-S} + Q_\lambda^{-H} .$$

- For $\lambda^\pm = 0$, i.e., $\lambda = \mathcal{O}(1/r)$, one sees that $Q_\lambda^\pm = 0$. Trivial gauge transformations.
- For $\lambda^\pm \neq 0$, one gets charges of global symmetries. What is the interpretation of these charges?

Asymptotic symmetries

- The hard part is easy:

$$Q_\lambda^{+H} = \int_{\mathcal{I}^+} \mathrm{d}u\, \mathrm{d}^2z\, \sqrt{\gamma} r^2 \lambda^+ j_u\,, \qquad Q_\lambda^{-H} = \int_{\mathcal{I}^-} \mathrm{d}v\, \mathrm{d}^2z\, \sqrt{\gamma} r^2 \lambda^- j_v\,,$$

correspond to integrals of distributions of outgoing/incoming charges, weighted with $\lambda^\pm(z,\bar{z})$ over $\mathcal{I}^\pm$.

- The soft parts can be written as

$$Q_\lambda^{\pm S} = \int_{S^2} \mathrm{d}^2z \sqrt{\gamma} \gamma^{AB} N_A^\pm \partial_B \lambda^\pm\,, \qquad \text{with} \quad N_A^+ = \int_{\mathcal{I}_{\hat{n}}^+} \mathrm{d}u\, F_{uA}\,, \qquad N_A^- = \int_{\mathcal{I}_{\hat{n}}^-} \mathrm{d}v\, F_{vA}\,.$$

- $N_A^\pm$ is referred to as the "soft photon mode". Reason, as the $\omega \to 0$ limit of the Fourier transform

$$N_A^+ = \lim_{\omega \to 0} \int_{-\infty}^{+\infty} \mathrm{d}u\, \mathrm{e}^{\mathrm{i}\omega u}\, F_{uA}\,, \qquad N_A^- = \lim_{\omega \to 0} \int_{-\infty}^{+\infty} \mathrm{d}v\, \mathrm{e}^{\mathrm{i}\omega v}\, F_{vA}\,,$$

it gives creation and annihilation operators for outgoing/ingoing photons of energy $\omega \to 0$, upon canonical quantization.

From asymptotic symmetries to soft photons

- Consider a collection of N charges with initial velocities $\vec{v}_k^{\,\text{in}}$ and final velocities $\vec{v}_k^{\,\text{out}}$ $(k = 1, \cdots, N)$. If $\vec{v}_k^{\,\text{in}} = \vec{v}_k^{\,\text{out}}$ for all k, then antipodal matching implies that $Q_\lambda^{+H} = Q_\lambda^{-H}$, so

$$Q_\lambda^{+H} - Q_\lambda^{-H} = \begin{cases} 0 & \text{if } \vec{v}_k^{\,\text{in}} = \vec{v}_k^{\,\text{out}} \text{ for all } k \\ \mathcal{O}(1) & \text{if } \vec{v}_k^{\,\text{in}} \neq \vec{v}_k^{\,\text{out}} \text{ for some } k \end{cases}$$

- One has that

$$F_{uA} \sim \mathcal{O}(1/r) \;\text{ for a Coulomb field} \qquad\qquad F_{uA} \sim \mathcal{O}(1) \;\text{ for radiation}\,.$$

So

$$Q_\lambda^{\pm S} = \begin{cases} 0 & \text{if no radiation passes through } \mathcal{I}^\pm \\ \mathcal{O}(1) & \text{if radiation passes through } \mathcal{I}^\pm \end{cases}$$

- Suppose $Q_\lambda^{-S} = 0$ (no incoming radiation). Then soft charge conservation implies that

$$Q_\lambda^{+S} = -\left(Q_\lambda^{+H} - Q_\lambda^{-H}\right) = \mathcal{O}(1) \qquad\qquad \text{if } \vec{v}_k^{\,\text{in}} \neq \vec{v}_k^{\,\text{out}} \text{ for some } k\,.$$

So, if there is acceleration, soft radiation passes through $\mathcal{I}^+$. Bremsstrahlung can be viewed as a consequence of soft charge conservation.

From asymptotic symmetries to the soft photon theorem

- Are asymptotic symmetries really symmetries? Do the soft charges satisfy a Ward identity?

$$\langle \text{out} \,|\, Q^+_\lambda S - S Q^-_\lambda \,|\, \text{in} \rangle \overset{?}{=} 0\,, \quad \text{with}$$
$$|\text{in}\rangle = |p_1^{\text{in}}, \cdots, p_n^{\text{in}}\rangle\,, \qquad |\text{out}\rangle = |p_1^{\text{out}}, \cdots, p_m^{\text{out}}\rangle\,.$$

- Note that

$$Q^-_\lambda\, |\text{in}\rangle = \int_{S^2} \mathrm{d}^2 z \sqrt{\gamma}\gamma^{AB}\partial_A \lambda^- N^-_B\, |\text{in}\rangle + \sum_{k=1}^{n} Q_k^{\text{in}} \lambda(z_k^{\text{in}}, \bar{z}_k^{\text{in}})\, |\text{in}\rangle\,,$$
$$\langle \text{out}|\, Q^+_\lambda = \int_{S^2} \mathrm{d}^2 z \sqrt{\gamma}\gamma^{AB}\partial_A \lambda^+ \,\langle \text{out}|\, N^+_B + \sum_{l=1}^{m} Q_l^{\text{out}} \lambda(z_l^{\text{out}}, \bar{z}_l^{\text{out}})\, \langle \text{out}|\,.$$

So the Ward identity is equivalent to the soft photon theorem:

$$\underbrace{\int_{S^2} \mathrm{d}^2 z \sqrt{\gamma}\partial^A \lambda^+ \left\langle \text{out} \,\middle|\, N^+_A S - S N^-_A \,\middle|\, \text{in} \right\rangle}_{\left\langle \text{out} \,\middle|\, a_+^{\text{out}}(q) S \,\middle|\, \text{in} \right\rangle \text{ in soft limit}} \overset{!}{=} \underbrace{\left[\sum_{k=1}^{n} Q_k^{\text{in}} \lambda(z_k^{\text{in}}, \bar{z}_k^{\text{in}}) - \sum_{l=1}^{m} Q_l^{\text{out}} \lambda(z_l^{\text{out}}, \bar{z}_l^{\text{out}})\right]}_{\text{Low-Weinberg soft factor}} \langle \text{out} \,|\, S \,|\, \text{in} \rangle$$

Memory from symmetry

- The electromagnetic memory effect consists of a velocity kick given to a test particle with mass m and charge Q after the passage of an electromagnetic wave. Evaluated at $\mathcal{I}^+$ far away from the source of radiation:

$$\Delta\vec{v} = \frac{Q}{m}\int_{-\infty}^{+\infty} \mathrm{d}t\,\vec{E}\,, \qquad \int_{-\infty}^{+\infty} \mathrm{d}t\,\vec{E} \quad \to \quad \int_{-\infty}^{+\infty} \mathrm{d}u\,F_{uA}\,.$$

- Note that due to our boundary conditions:

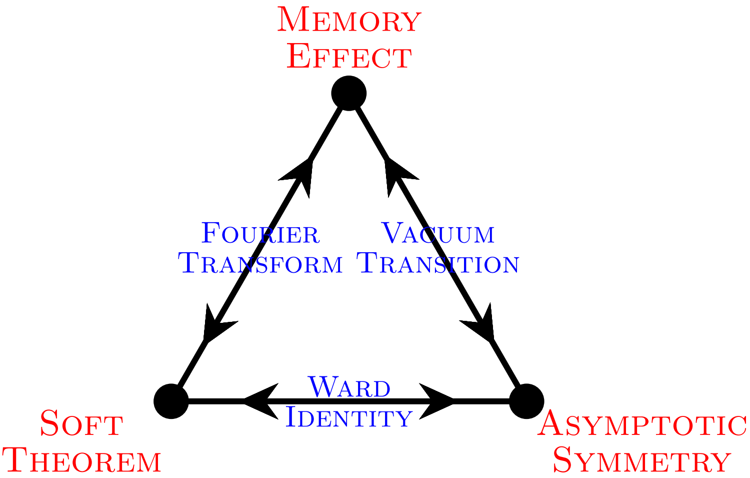
$$\int_{-\infty}^{+\infty} \mathrm{d}u\,F_{uA} = \int_{-\infty}^{+\infty} \mathrm{d}u\,\Big(\partial_u \underbrace{A_A}_{\mathcal{O}(1)} - \partial_A \underbrace{A_u}_{\mathcal{O}(1/r)}\Big) = A_A^{(0)}(u=+\infty) - A_A^{(0)}(u=-\infty)\,.$$

Due to the finite energy requirement $A_A^{(0)}(u=-\infty)/A_A^{(0)}(u=+\infty)$ only depend on the initial/final velocities of the charges that source the electromagnetic wave. Moreover

$$A_A^{(0)}(u=\pm\infty) = \partial_A\phi_\pm \qquad \longrightarrow \qquad \int_{-\infty}^{+\infty} \mathrm{d}u F_{uA} = \partial_A\left(\phi_+ - \phi_-\right)\,.$$

- The memory effect is an asymptotic symmetry! Asymptotic symmetries are measurable!

Conclusions: an infrared triangle



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